1. (a) Define what is the extended transition function of a DFA.
(b) Convert the NFA on the right into an equivalent DFA.

2. Find regular expressions which describe the languages below
(a) \( L_1 = \{ \alpha \in \{a,b\}^*: \alpha \text{ contains both } aab \text{ and } bab \text{ as substrings} \} \)
(b) \( L_2 = \{ \beta \in \{0,1\}^*: \beta \text{ has at most one occurrence of } 01 \} \)

3. (a) Find all the inaccessible states in the DFA below.
(b) Then partition the remaining states into blocks of indistinguishable states and find the reduced machine.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>D</td>
<td>G</td>
<td>B</td>
<td>B</td>
<td>G</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>B</td>
<td>F</td>
<td>A</td>
<td>E</td>
<td>A</td>
<td>H</td>
</tr>
</tbody>
</table>

4. (a) Find a DFA which accepts precisely the strings in the language \( L_4 = \{ \alpha \in \{a,b\}^*: \lfloor n_0(\omega) - n_1(\omega) - 1 \rfloor \pmod{4} < 2 \} \).
(b) Verify your DFA works with babb as input.

5. (a) Define what are unreachable productions and non-terminating productions in a CFG \( G \).
(b) Find a context-free grammar which generates the language \( L_5 = \{ a^k b^n : k < n + 3 \} \cup \{ b^k a^n : k > 2n + 2 \} \).

6. Let \( A, B \) and \( C \) be languages based on the alphabet \( \{0,1\} \).
(a) Is it always true that \( (B \cdot A^k) \cup (C \cdot A^k) \subseteq (B \cup C) \cdot A^k \)?
(b) Is it always true that \( (B \cdot A^k) \cap (C \cdot A^k) \subseteq (B \cap C) \cdot A^k \)?
Justify your answers completely.
1(a) The extended transition function of a DFA is defined recursively by 
\[ \delta^*(q, \lambda) = q \] and 
\[ \delta^*(q, \varphi a) = \delta(\delta^*(q, \varphi), a) \] for any \( a \in \Sigma \).

(b) \[ R(\lambda) = \{ C \} \] 
\[ \emptyset \overset{0}{\rightarrow} \{ B \} \] 
\[ \{ A, C \} \overset{0}{\rightarrow} \emptyset \] 
\[ \emptyset \overset{1}{\rightarrow} \emptyset \] 
\[ \emptyset \overset{1}{\rightarrow} \emptyset \]

The DFA is

2(a) \[ \cdots aab-bab\cdots, \cdots bab-aab\cdots, \cdots aabab\cdots \]

\[ E_1 = (a+b)^* (aab, (a+b)^*, bab-bab, (a+b)^* aab + aabab) (a+b)^* \]

(b) Two cases: no 01's and one 01

\[ E_2 = 1^* 0^* + 1^* 0^* 0^* (01). 1^* 0^* \]

3(a) \[ \rightarrow D \overset{0}{\rightarrow} B \overset{0}{\rightarrow} G \overset{0}{\rightarrow} B \]
\[ \overset{1}{\rightarrow} H \overset{1}{\rightarrow} B \overset{0}{\rightarrow} F \overset{0}{\rightarrow} A \]
\[ \overset{1}{\rightarrow} D \overset{1}{\rightarrow} C \overset{1}{\rightarrow} B \overset{1}{\rightarrow} G \]
\[ \overset{1}{\rightarrow} E \overset{1}{\rightarrow} H \]
\[ \vdots \quad : \quad E \text{ is inaccessible} \]

(b) \( P_0 : \{ C, D, G, H \} \text{ \{A, B, F\} } M_R = \)
\( P_1 : \{ C, D \} \text{ \{G, H\} \text{ \{A, B, F\} } \)
\( P_2 : \{ C, D \} \text{ \{G, H\} \text{ \{A, F\} \{B\} } \)
\( P_3 : \{ C, D \} \text{ \{G\} \text{ \{H\} \text{ \{A, F\} \{B\} } \)
\( P_4 : \{ C, D \} \text{ \{G\} \text{ \{H\} \text{ \{A, F\} \{B\} } = P_3. \)
4(a) Let \( f(\omega) = \eta_0(\omega) - \eta_1(\omega) - 1 \pmod{4} \) and \( A_i (i = 0, 1, 2, 3) \) keep track of \( f(\omega) \) where \( \omega \) is a part of the string processed. Then \( f(\lambda) = 0 - 0 - 1 = 3 \pmod{4} \). So \( A_3 \) will be the accepting state. Also \( f(\omega) < 2 \) when \( f(\omega) = 0 \) or \( f(\omega) = 1 \), so \( A_0 \) and \( A_1 \) will be the accepting states. Finally
\[
\begin{align*}
\Delta(wa) &= \eta_0(\omega a) - \eta_1(\omega a) - 1 = \eta_0(\omega) - \eta_1(\omega) - 1 - 1 = f(\omega) - 1 = f(\omega) + 3 \\
\Delta(wb) &= \eta_0(\omega b) - \eta_1(\omega b) - 1 = \eta_0(\omega) + 1 - \eta_1(\omega) - 1 = f(\omega) + 1 \\
\end{align*}
\]

(b) Let \( \omega = babb \)

Input: \( b \ a \ b \ b \)

States: \( A_0 \ A_1 \ A_3 \ A_0 \ A_1 \)

\( f(\omega) = 1 \) according to DFA

Check:
\[
\eta_0(\omega) - \eta_1(\omega) - 1 = 3 - 1 - 1 = 1 \pmod{4}
\]

5(a) An unreachable production of \( G \) is any production which contains a variable that cannot be reached from the starting variable \( S \). A non-terminating production is any production that contains a variable which does not terminate or lead to something that eventually terminates.

(b) \( S \rightarrow S_1 | S_2 \)  

(This gives the union from \( S_1 \& S_2 \))

\[
\begin{align*}
S_1 &\rightarrow AS_1b | AA, \ A \rightarrow a/\lambda \\
S_2 &\rightarrow b S_2 a | Bb \quad \ B \rightarrow bB | bbb \\
\end{align*}
\]

\[
\begin{align*}
S_1 &\Rightarrow AS, b \Rightarrow AAS, b \Rightarrow \ldots \Rightarrow A^n S_b^n \Rightarrow A^n, AAb^n = A^{n+2}b^n \\
The \ A's \ can \ be \ replaced \ by \ a \ or \ \lambda \ to \ give \ a^k b^n \\
\text{with } k \leq n+2. \ \text{So } S_1 \text{ generates } \{a^k b^n \mid k \leq n+3\}. \\
S_2 &\Rightarrow b S_2 a \Rightarrow b^4 S_2 a^2 \Rightarrow \ldots \Rightarrow b^n S_2 a^n \Rightarrow b^n B a^n = b^n b^2 a^n \\
\Rightarrow \ldots \Rightarrow b^n b^6 B a^n = b^n b^{n+6} a^n = b^{2n+6} a^n = b^{k-} a^n \\
\text{with } k \geq 2n+3. \ \text{So } S_2 \text{ generates } \{b a^k a^n \mid k > 2n+2\}\]
(a) \( \text{YES. Let } \phi \in (B.A^R) \cup (C.A^R) \). Then
\( \phi \in B.A^R \) or \( \phi \in C.A^R \). Hence either
\( \phi = \beta.1^R \) with \( \beta \in \mathbb{B} \) and \( \alpha \in \mathbb{A} \), or
\( \phi = \gamma.1^R \) with \( \gamma \in \mathbb{C} \) and \( \alpha \in \mathbb{A} \).
In the first case \( \phi = \beta.1^R \in (B.1^R) \), \( \mathbb{A}^R \) because \( \beta \in (B.1^R) \)
and \( \alpha \in \mathbb{A} \). And in the second case \( \phi = \gamma.1^R \in (C.1^R) \), \( \mathbb{A}^R \) because \( \gamma \in (C.1^R) \) and \( \alpha \in \mathbb{A} \). So in
either case \( \phi \in (B.1^R) \). \( \mathbb{A}^R \). Hence we will
always have \( (B.A^R) \cup (C.A^R) \subseteq (B.1^R) \).

(b) \( \text{NO. Let } A = \{1, 10\}, \quad B = \{10\}, \quad \text{and } C = \{1\}. \)
Then \( (B.1^R) = \emptyset \), so \( (B.1^R) \). \( \mathbb{A}^R = \emptyset \). \( \{1, 01\} \) = \( \emptyset \)
Also \( B.1^R = \{10, 1, 100^R = \{10, \{1, 01\} = \{101, 100\}\} \)
and \( C.1^R = \{1\}, \{1, 01\} = \{11, 101\}\)
\( \therefore (B.1^R) \cap (C.1^R) = \{101, 100\} \cap \{11, 101\} = \{101\} \).
Hence in this particular case
\( (B.1^R) \cap (C.1^R) \neq (B.1^R) \).
So it is not always true that for all \( \mathbb{A}, \mathbb{B}, \mathbb{C} \)
\( (B.A^R) \cup (C.A^R) \subseteq (B.1^R) \). \( \mathbb{A}^R \)