(10) 1(a) If L is any language on \{a, b\}, we always have \((L^c)^c = (L^c)^c\).
(b) The set of all regular expressions on \{0, 1\} is countable.
(c) If L is any non-empty language, L* will always be infinite.
(d) If a DFA M has at least one accepting state, then \(L(M) \neq \emptyset\).
(e) If G is a CFG with a production of the form \(S \rightarrow ASb\) and G has no useless production, then \(L(G)\) is infinite.

Just write down the correct answer.

(18) 2(a) Find a regular expression \(E\) for the set of all strings in \(\{0, 1\}^*\) which contains at most one occurrence of the string 00.
Ans: \(E = \)

(b) If \(G = \{S \rightarrow ASBB, S \rightarrow b, A \rightarrow a, B \rightarrow b, B \rightarrow \lambda\}\), then
\(L(G) = \)

(c) If M is the NFA below, then

```
A \rightarrow b \rightarrow B \rightarrow C \leftarrow a \rightarrow C
```

\(L(M) = \)

(d) Find a RLG G for \(b^*a^*b^*a^*\).
Ans: \(G = \)

(e) Find a DFA M with \(L(M) = (0, 1)^* + (1)^*\).
Ans: \(M = \)

Use the back of this paper for question #3.

(12) 3(a) Define what are useless productions in a context-free grammar G.
(b) Define what is a leftmost derivation in a context-free grammar G.
(c) Define when two states A and B of a DFA M are indistinguishable.
(d) Define the extended transition function of an NFA and specify its domain and co-domain.
1. (a) FALSE. Take $L = \emptyset$. Then $\lambda \in (L^c)^*$ but $\lambda \notin (L^*)^c$.
   (b) TRUE. The set of regular expressions $\{ \lambda, 0, 1, \emptyset, +, \cdot, *, (\cdot)^* \}$
   (c) FALSE. Take $L = \{ \lambda \}$. Then $L^* = \{ \lambda \}$ which is finite.
   (d) FALSE. Consider the DFA
       \[
       \begin{array}{c}
       A \rightarrow \{0, 1\} \rightarrow B \rightarrow \{0, 1\} \rightarrow C
       \end{array}
       \]
   (e) TRUE. $S \rightarrow ASB$ will generate $A^nSB^n$ which will always terminate.

2. (a) $E = (1+01)^* (\lambda+0) + (1+01)^* 00 (1+10)^*$
   (b) $L(G) = \{a^n b^{k+1} : 0 \leq k \leq 2n \text{ and } n \geq 0 \}$
   (c) $L(M) = b. (b^* a)^* + b. c. (b + ac)^*$
   (d) $G: S \rightarrow bS, S \rightarrow A, A \rightarrow aA, A \rightarrow bB, B \rightarrow aB, B \rightarrow \lambda$

3. (a) An unreachable production is one which involves a variable that cannot be reached from the starting variable of $G$. A non-terminating production is one which contains a variable that does not eventually terminate into terminal symbols. These are the two kinds of useless productions.
   (b) A leftmost derivation is one in which the leftmost variable is replaced at each step of the derivation.
   (c) Two states $A$ and $B$ in a DFA $M$ are indistinguishable if for each string $w \in \Sigma^*$, $\delta^*(A, w) \in A(M) \iff \delta^*(B, w) \in A(M)$.
   (d) The extended transition function of an NFA $M$ is defined by $\Delta^*(p, w) = \{q \in Q : w \text{ can lead you from } p \text{ to } q \text{ in } M\}$
   domain($\Delta^*$) = $Q \times \Sigma^*$, codomain($\Delta^*$) = $P(Q)$. 