Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

(15) 1. (a) Define what is a regular expression over the alphabet \{a, b, c\}.

(b) Convert the NFA on the right into an equivalent DFA.

(15) 2. Find regular expressions which describe the languages below.
(a) \(L_1 = \{\alpha \in \{0, 1\}^* : \alpha \text{ contains both } 110 \text{ & } 101 \text{ as substrings}\}
(b) \(L_2 = \{\beta \in \{0, 1\}^* : \beta \text{ has exactly two occurrences of } 00\}

(20) 3. (a) Find all the inaccessible states in the DFA below.
(b) Then partition the remaining states into blocks of indistinguishable states and find the reduced machine.

\[
\begin{array}{cccccccc}
A & B & \rightarrow & C & D & E & F & G & H \\
0 & H & G & B & C & B & G & A & B \\
1 & D & B & D & A & G & H & E & A \\
\end{array}
\]

(15) 4. Find a DFA which accepts precisely the strings in the language \(L_4 = \{\omega \in \{a, b\}^* : f(\omega) < 2\}\), where \(f(\omega) = [2n_a(\omega) - 3n_b(\omega) - 1] \mod 4\), and then check your DFA with abbab as input.

(20) 5. (a) Define what are useless productions of a context-free grammar \(G\).
(b) Find a context-free grammar which generates the language \(L_5 = \{a^k b^{n+1} : k > 2n\} \cup \{a^k c^n : k < 3n+2\}\).

(15) 6. Let \(A, B, \) and \(C\) be languages based on the alphabet \(\{0, 1\}\).
(a) Is it always true that \((A-B) \cup C \subseteq (A \cup C) - (B \cap C)\)?
(b) Is it always true that \((A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)\)?
Justify your answers completely.
1. (a) A regular expression over \{a, b, c, \lambda\} is defined recursively as follows: (i) \(a, b, c, \lambda\), and \(\emptyset\) are regular expressions. (ii) If \(E\) and \(F\) are regular expressions, then so are \((E+F), (E.F), \lambda\) and \(E^*\).

(b) 
\[
\begin{align*}
A & \rightarrow a, b, c, A, a, b, c, a, b, c \\
B & \rightarrow a, b, c, \lambda, a, b, c, a, b, c \\
C & \rightarrow a, b, c, a, b, c, a, b, c, \lambda
\end{align*}
\]

2. (a) 
\[
\begin{align*}
& \cdots 110 \cdots 101 \cdots 110 \cdots, \quad \cdots 1101 \cdots 10110 \cdots \\
& (0+1)^* (110(0+1)^* 01 + 101(0+1)^* 10 + 1101 + 10110)(0+1)^* \end{align*}
\]

(b) 
\[
\begin{align*}
& \cdots 110 \cdots 101 \cdots 110 \cdots, \quad \cdots 1101 \cdots 10110 \cdots \\
& (0+1)^* (110(0+1)^* 01 + 101(0+1)^* 10 + 1101 + 10110)(0+1)^* \end{align*}
\]

3. (a) 
\[
\begin{align*}
A & \rightarrow a, B, C, D \\
B & \rightarrow a, \lambda, B \\
C & \rightarrow a, B, C, D \\
D & \rightarrow a, B, \lambda, D \\
E & \rightarrow a, B, C, \lambda \\
F & \rightarrow a, B, C, \lambda, B, C, \lambda
\end{align*}
\]

(b) 
\[
P_0: \{A, B, D\} \{C, E, F, G, H\}
\]

4. (a) Let \(A_i (i=0, 1, 2, 3)\) keep track of the fact that \(f(n) \equiv i \pmod{4}\). Since \(f(0) = 2n_0(0) - 3n_0(0) - 1 = 0 - 0 - 1 = 3 \pmod{4}\), \(A_3\) will be the initial state. Also, since \(0 < 2\) and \(1 < 2\), \(A_0\) & \(A_1\) will be the accepting states.
4(a) \[ f(wa) = 2\eta_a(wa) - 3\eta_b(wa) - 1 = 2\eta_a(w) - 3\eta_b(w) - 1 + 2 \]
\[ = f(w) + 2 \pmod{4} \]
\[ f(wb) = 2\eta_a(wb) - 3\eta_b(wb) - 1 = 2\eta_a(w) - 3\eta_b(w) - 1 - 3 \]
\[ = f(w) - 3 \pmod{4} \]
\[ = f(w) + 1 \pmod{4} \]
\[ f(abab) = 2(2) - 3(3) - 1 \]
\[ = -6 \pmod{4} = 2 \pmod{4} \]

5(a) A useless production is an unreachable or non-terminating production. An unreachable production is one which involves a variable which cannot be reached from the starting variable. A non-terminating production is one which involves a variable which does not eventually terminate.

(b) \[ S \rightarrow A | B \]
\[ A \rightarrow bCaAb | Cb, \quad C \rightarrow aC | \lambda, \quad \{a^k b^n c^m : k \geq 2n + 1\} \]
\[ B \rightarrow DDDDBc | Da, \quad D \rightarrow a | \lambda, \quad \{a^k c^n : k \leq 3n + 1\} \]
\[ S \rightarrow A \Rightarrow CaAb \Rightarrow \cdots \Rightarrow C^2Ab^n \Rightarrow C^2Cb_b^n \Rightarrow \cdots \Rightarrow C^{2n+1} b^{n+1} \]
\[ S \rightarrow B \Rightarrow a^3Bc \Rightarrow \cdots \Rightarrow a^3Bc^n \Rightarrow a^3Da_c^n \Rightarrow a^3c^{n+1} \cdot a \cdot c^n \]

6(a) No. Let \( A = \{1\}, \ B = \{\lambda\} \) and \( C = \{\lambda, 1\} \). Then \( (A - B, C) = (\{1\}, \{1\}) \{/1, 1, \} = \{1\}, \{\lambda, 1\} = \{\lambda, 1\} = \{\lambda, 1\} \) and
\( (A, C) \cap (B, C) = \{1, \lambda, 1\} - \{\lambda, 1\} = \{1, 1\} - \{\lambda, 1\} = \{1\} \).
\( \therefore (A - B, C) \neq (A, C) \cap (B, C) \) in general.

(b) Yes. Let \( y \in (A\cap B, C) \). Then \( y = x \cdot y \) with \( x \in A \cap B \) and \( yeC \). Since \( x \in A \cap B \), \( x \in A \) and \( x \in B \).
\( \therefore y = x \cdot y \) with \( x \in A \) and \( yeC \). Hence \( y \in A \cdot C \) and \( y \in B \cdot C \).
Thus \( y \in (A \cdot C) \cap (B \cdot C) \). Hence \( (A \cap B) \cdot C \subseteq (A \cdot C) \cap (B \cdot C) \).