STA-2122  Sample Test 2

I) identify the given variables as being discrete or continuous.

a. The pressure at which water is being drawn from a well into a summer cottage.
b. The heart rate (number of beats per minute) of an American adult
c. The length of time an employee is late for work.
d. The market value of a publicly listed security on a given day.
e. The amount of flu vaccine in a syringe.
f. The weight (in pounds) of a food item bought in a supermarket.
g. The number of voters in a sample of 500 who favor the impeachment of president

II) A discrete random variable X can assume the five possible values 2, 3, 5, 8, and 10. Its probability distribution is given below:

<table>
<thead>
<tr>
<th>X</th>
<th>P (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Find the following probabilities

a. P (8)?
b. What is the probability that x equals 2 or 10?
c. What is P (X < 5)?
d. Find P (X < 3 \cup X > 8).
e. Calculate the mean, \( \mu \) and standard deviation, \( \sigma \) of random variable X.

III) The probability that randomly selected flight will arrive on time is 0.7230. The table gives all possible values of random variable X: the number of flights arriving in time out of 6 randomly selected flights with corresponding probabilities.

<table>
<thead>
<tr>
<th>X</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.005</td>
</tr>
<tr>
<td>1</td>
<td>0.0071</td>
</tr>
<tr>
<td>2</td>
<td>0.0462</td>
</tr>
<tr>
<td>3</td>
<td>0.1607</td>
</tr>
<tr>
<td>4</td>
<td>0.3145</td>
</tr>
<tr>
<td>5</td>
<td>0.3283</td>
</tr>
<tr>
<td>6</td>
<td>0.1428</td>
</tr>
</tbody>
</table>

1. Find the probability that at least 5 flights arrive on time.
2. Find the probability that at most 2 flights arrive on time.
3. Find the probability that more than one flight arrives on time.
4. Find the probability that at least one flight arrives on time.

IV) Let x be a binomial random variable. The probability, p of a success on a single trial is 0.6 and the number of trials, n are 15. Using the binomial table

1. Find \( P(X \leq 5) \)
2. Find \( P(3 \leq X < 8) \)
3. Find \( P(X = 2) \)

V) The State high Way Patrol has determined that one out of every 10 calls for help originating from roadside call boxes is a hoax. Five calls for help have come in and five trucks have been dispatched. Let X be the number of calls that were hoaxes.

a. What is the name of the probability distribution of X- the number of calls that were hoaxes? Explain
b. Give the formula of the probability distribution of X
c. What is the probability that none of the calls was a hoax?
d. What is the probability that only three of the callers really needed the assistance?

VI. A machine has 10 identical components which function independently. The probability that a component will fail is 0.2. The machine will stop working if more than 3 three components fail.
1. Find the probability that the machine will be working.
2. Find mean and standard deviation of $X$ - the number of components that fail.

VII) The director of urban planning program at Columbia University estimates (Los Angeles Time article) that there are 21,000 unlicensed cabs in Manhattan. The unlicensed cabs are twice the number of legal cabs. On a particular evening, 60 customers independently hail a cab in Manhattan. Let $x$ be the number of unlicensed cabs that respond.

a. What is the name the probability of distribution of $X$?
b. What are the values of parameters of this probability distribution?
c. Calculate the expected value of $X$
d. Calculate the standard deviation of $X$.

VIII) Let $Z$ be a standard normal variable.

Sketch a figure for each problem and find the corresponding probabilities.
1. Find $P (0 < Z < 1.6)$.
2. Find $P (Z < -1.79)$.
3. Find $P (-1.47 < Z < 2.52)$.
4. Find $P (1.56 < X < 2.33)$
5. Find $P (X > 1.96)$
6. Find $P (X < -2.56)$

Find a number $Z_0$ such that

$P (-Z_0 < Z < Z_0) = 0.8740.$
$P (Z < Z_0) = 0.3740.$

IX) The length of life of a certain type of automatic washer is approximately normally distributed, with a mean of 3.1 years and a standard deviation of 1.2 years. What percentage of automatic washers are
a. Between 2.8 and 4.7 years
b. Less than 4.5 years.
c. If this type of washer is guaranteed for 1 year, what fraction of original sales will require replacement?
d. What should be life time $x_0$ such that only 2% of the washers have the life time less than $x_0$?

X) A study of the amount time it takes a mechanic to rebuild the transmission for a 1992 Chevrolet Cavalier shows that the mean is 8.4 hours and standard deviation is 1.8 hours. If 40 mechanics are randomly selected,

a) Compute mean and standard deviation of the sampling distribution of $\bar{x}$ - the mean rebuild time of the sample of 40 sampled mechanics.

b) What is the name of the sampling distribution of $\bar{x}$? Sketch the distribution
c) What is the approximate probability that the mean rebuild time ($\bar{x}$) of 40 mechanics exceeds 7.7 hours.
d) What is the probability that the mean rebuild time is between 10.2 and 12.00