# $\chi^2/\nu$: A measure of quality of a fit.

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One way to determine if a set of data is well represented by a fitting function is to calculate the chi-squared per degree of freedom (also know as the reduced chi-squared):

$$\frac{\chi^2}{\nu} = \frac{1}{\nu}\sum_i^n \left[\frac{f(x_i) - y(x_i)}{\sigma_i}\right]^2, \tag{1}$$

where $f(x_i)$ is the value of the fitting function at a given value of $x_i$, $y(x_i)$ is the measured result at $x_i$, and $\sigma_i$ is the uncertainty in the measured value at $x_i$. In the case of the Poisson (or Gaussian) fit, $y(x_i)$ will be the number of occurrences in a particular bin of $x_i$ and $\sigma_i$ will be the square-root of the number of occurrences. So if you found ten counts at a rate of 50 Hz, $y(50) = 10$ and $\sigma = \sqrt{10}$.

In the above equation, $\nu$ represents the number of degrees of freedom. This will depend on the number of data points you fit and the number of parameters in your fitting function. For example, if you have 15 bins that you are fitting and you are fitting with a Gaussian, which has three parameters (amplitude, mean, and standard deviation), then $\nu = 15 - 3 = 12$. A Poisson fit has two parameters (amplitude and mean) so $\nu = 13$.

So what do we expect for $\chi^2/\nu$? If you take the square-root of this, a "good" fit should have $\sqrt{\chi^2/\nu} \approx 1$. A "bad" fit will have a value significantly larger than one–say something like 2. A value significantly less than one–say 0.5, or so–has several different possible meanings. One thing it could mean is that the uncertainties are over-estimated. Sometimes in experimental measurements, uncertainties will have components that are due to estimated systematic uncertainties. It is frequently the

case that an experimentalist will over estimate these quantities to "be on the safe side" (something good experimentalist avoid doing). This is not likely to be the case for our rate measurements because our uncertainties are simply the square root of the number of occurrences. A second reason for a small reduced chi-squared is that statistical fluctuations simply have lead to good agreement between data and the fitting function. It is also possible that a large reduced chi-squared could be the result of statistical fluctuations. A third reason for very low reduced chi-squared is that the fitting function has too many parameters–you are essentially connecting the dots. This won't be our case since one of our fitting functions MUST be an accurate representation of this type of data. This case usually arises when one is trying to fit data with a polynomial and the fit has too many orders.

In the end, one can determine the probability that the value of the reduced chi-squared is lower or greater than the measured value. There are tables in Bevington–as well as various web-based programs–that will give these probabilities. A high probability (say bigger than 60%) of exceeding a certain reduced chi-squared means that the fit is probably "too good" while a low probability (say less than 40%) means that the fit is "bad". Generally, the probability of exceeding a reduced chi-squared of one is 50%. This procedure of measuring the probability of exceeding a certain value of chi-square is known as a "chi-square" test. We aren't going to do this at this time but check it out if you are curious.

There are lots of other means for determining how good the fit is. These include the "f-test" and determining quantities such as the "confidence" levels. Again, we aren't going to worry about this for now but you should check it out if you are curious.

If this seems like there is a lot of uncertainty left in the interpretation, don't feel bad. What looks like a good fit to one person may look bad to another. There is frequently a lot of ambiguity when one sits down to judge the quality of a fit. For our purposes, the best thing–and the easiest thing–to do is to look at $\sqrt{\chi^2/\nu}$. This will generally be a pretty good measure of fit quality.