Section I—Theory
Chapter 1

Risk and Model Fundamentals

- Introduction
- Risk Measures
- Measuring Performance Vs. a Benchmark
- Portfolio Management Strategies and Risk
- Multiple-Factor Models
Superior investment performance is the product of careful attention to four elements:

- Forming reasonable return expectations.
- Controlling risk so that the pursuit of opportunities remains tempered by prudence.
- Controlling costs so that investment profits are not dissipated in excessive or inefficient trading.
- Controlling and monitoring the total investment process to maintain a consistent investment program.

These four elements are present in any investment management problem, be it a strategic asset allocation decision, an actively managed portfolio, or an index fund — managed bottom-up or top-down, via traditional or quantitative methods.

Figure 1: The Performance Pyramid
Superior investment results depend on understanding and controlling risk. Barra models and software permit you to minimize risk within your constraints, and to balance your pursuit of return with acceptable risk. In this Handbook we discuss how Barra models are put together, how risk is optimally controlled, and how a risk model can contribute to a deeper understanding of investment performance.

Risk Measures

A definition of risk that meets the criteria of being universal, symmetric, and flexible is the standard deviation of return.  

Standard Deviation

A portfolio’s future return is described by a probability distribution. Every return has a probability of occurring, with one expected return having the largest chance of being realized. The width of this probability distribution represents risk to an investor. This is the uncertainty associated with the portfolio’s future value.

The standard deviation (\(\sigma\)) can be used to quantify the width of a probability distribution and describe the uncertainty, variability, or risk associated with the return of an asset or a portfolio.

\[ 1 \text{ An economist would call the standard deviation a measure of uncertainty rather than risk.} \]
If $R_p$ is a portfolio’s total return, then the portfolio’s standard deviation of return is denoted by

$$\sigma_p \equiv \text{Std}[R_p]$$

A portfolio’s excess return ($r_p$) differs from the total return ($R_p$) by a constant $R_F$, so the risk of the excess return is equal to the risk of the total return. We typically quote this risk, or standard deviation of return, on an annualized basis. We also occasionally refer to this quantity as volatility.\(^2\)

The rough interpretation of standard deviation is that an asset return will be within one standard deviation of its expected value two-thirds of the time and within two standard deviations 19 times out of 20. Figure 3 on page 7 illustrates this idea.

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For example, suppose that we have determined that the standard deviation of an asset’s return is 3% and that its expected return is 2%. Using the normal distribution, we can determine that there is roughly a two-third probability that the asset will return between -1% and 5% in the next year; a 95% likelihood that it will return between -4% and 8%; and a 5% chance that it will return less than -4% or greater than 8%.

There is considerable evidence that financial returns are not normal. In particular, extreme values occur more often than predicted by a normal distribution. Still, at the levels of one or two standard deviations, financial returns can be treated as approximately normal. Assuming a normal distribution of portfolio or asset returns enables us to assign explicit probabilities to most reasonable returns.

**Variance**

One disadvantage of the standard deviation is that it is not additive, that is, the standard deviation of a portfolio is not simply the weighted average of the standard deviations of the individual assets in the portfolio.
Fortunately, the square of standard deviation, *variance* ($\sigma^2$), can be used to add sources of risk together, as in the case of adding the risks of individual assets to determine the risk of a portfolio.

The formulae are

$$\sigma = \text{Std}[r] = \sqrt{\text{Var}[r]} \quad \text{Eq. 1}$$

$$\sigma^2 = \text{Var}[r] = E[(r - \bar{r})^2] \quad \text{Eq. 2}$$

where

- $r$ = return
- $\bar{r}$ = expected or mean return
- $\text{Std}[x]$ = standard deviation of $x$
- $\text{Var}[x]$ = variance of $x$
- $E[x]$ = expected value of $x$

The standard deviation is the more common risk indicator since it is measured in the same units as return. Of course, if the standard deviation is known, the variance can be easily computed and vice versa.

**Other Measures of Risk**

*Value-at-risk* (VaR) characterizes the potential loss in a portfolio over a given time period for a chosen probability level. A typical statement of VaR would be “with 2.5% probability, losses will exceed 4%.”

*Beta* ($\beta$) captures the sensitivity of a portfolio to market movements. Typically, higher beta portfolios or assets tend to exhibit greater volatility.
Coefficient of determination, $R^2$, estimates the amount of variability in a portfolio that is explained by the market’s volatility. It is directly related to beta.

For fixed-income assets, duration measures the price responsiveness of an interest-sensitive asset to changes in interest rates. Duration is a reasonably good predictor as long as the change in interest rates is small and affects the entire yield curve. Macaulay duration describes the weighted average time to receipt of a series of cash flows, where the weights are the proportions of the present values of the cash flows to the present value of the entire series of cash flows.

As we will see in Chapter 3, “Fixed Income Multiple-Factor Modeling”, Barra employs principal component analysis to provide accurate measures for fixed-income risk.

Multiple-Factor Models

Multiple-factor models (MFMs) are formal statements about the relationships among asset returns in a portfolio. The basic premise of MFMs is that similar assets display similar returns. Assets can be similar in terms of fundamental company data (such as industry, capitalization, and indebtedness) or other attributes such as liquidity or duration.

MFMs identify common factors and determine return sensitivity to these factors. The resulting risk model describes a return as the weighted sum of common factor return and specific return, or that part of the return not attributed to model factors. In Barra’s MFMs, the risk profile responds immediately to changes in fundamental information. For more information, see “Multiple-Factor Models” later in this chapter.
Measuring Performance Vs. a Benchmark

Portfolio performance is most often measured with respect to a benchmark for investment performance and risk analysis. The benchmark is also known as the normal portfolio—that is, the asset basket a manager would hold in the absence of any judgmental information. It may reflect the manager’s particular style and biases.

While total risk describes the uncertainty of investment outcomes for an entire portfolio, active risk describes the risk that arises from the manager’s effort to outperform the benchmark. While outperforming the benchmark is the ultimate goal of an active manager, it is certainly not always achieved. Active risk (or tracking error) measures the uncertainty surrounding the portfolio’s performance with respect to its benchmark.

You may associate tracking error with passive management strategies and active risk with active management strategies. For the purposes of Barra’s analysis, these measures are considered the same.
Tracking Error and Active Risk

Active risk describes the expected volatility of the difference between the managed portfolio return and the benchmark return. They are both measured as the annualized standard deviation of active returns.

![Portfolio and Benchmark Returns](image)

*Figure 4: Portfolio and Benchmark Returns*

The difference between the portfolio’s and the benchmark’s returns is active return. The volatility of that return is active risk.

Portfolio Management Styles

Passive Management

In its broadest sense, passive management refers to any management strategy that does not rely on the possession of superior information. More specifically, disclosure of a passive investment strategy offers no competitive information that would undermine the strategy’s validity.

One type of passive management is indexing or tracking the performance of a particular index. An example is the “buy-and-hold” philosophy which exposes the portfolio only to benchmark risk. The second form of passive management is constructing a portfolio to match pre-specified attributes or constraints. The portfolio may be yield-biased with a selected beta or match an index within certain parameters. This is often called enhanced indexing.
Passive management procedures are distinguished by the following attributes:

- They exclude any transactions in response to judgments about security valuations and the market as a whole.
- They contain relatively minimal residual risk with respect to the benchmark or index.
- They often involve weighting by industry or sector.

**Active Management**

Active management refers to investment strategies designed to increase return by using superior information. In other words, the active manager seeks to profit from information that would lose its value if all market participants interpreted it in the same way. If, for example, an investment manager observed that securities with certain characteristics performed better (or worse) than expected, the manager could hold a larger (or smaller) proportion of that security to increase the subsequent value of the portfolio.

By following active management strategies, investors can add value to their portfolio if they predict returns better than the consensus expectations of the market. Information is obtained through ongoing research to forecast such things as yield curve changes, factor and industry returns, and macro-economic directions. At any given time, portfolio construction should reflect the tradeoff between risk and return—that is, any contribution to risk should be offset by the contribution to reward.

For a discussion of active investment strategies, see “Portfolio Management Strategies and Risk” on page 13.
Alphas

*Alpha* (α) generally refers to the expected exceptional return of a portfolio, factor, or individual asset. It measures how much an asset or portfolio has moved up or down during a given year. A positive alpha is the extra return that arises from taking specific (non-market) risk rather than passively taking market return. For example, an asset with an alpha of 1.40 means that, if the market and asset beta is 0, its price will rise by 40% in a year.

The use of alphas is a distinction of active management. It indicates that a manager believes a portion of expected return is attributable to particular factors, such as the rate of growth in earnings per share.

Portfolio Management Strategies and Risk

Equity Portfolio Strategies

For equity portfolios, the strategies include market timing, sectoral emphasis, and stock selection.

*Market timing* is the process of altering market risk exposure based on short-term forecasts in order to earn superior returns. The manager seeks to sell before the market goes down and buy before the market goes up. However, this strategy increases the variability in the portfolio beta, inducing increased systematic risk through time. Barra MFMs assist market timing by giving the investor a robust beta estimate for any portfolio and indicating the most efficient way to take on or reduce market risk, including the use of futures.
The second type of active management is *sectoral emphasis*. Sectoral emphasis can be thought of as a combination of the other active strategies. It is both factor timing and a broad version of stock selection. The manager attempts to increase residual return through manipulating common factor exposures. For example, the manager can bet on an industry of high-yield stocks. Because several sectors can be emphasized at any given time, diversification is possible. Barra MFMs possess detailed industry and risk index exposure information that can be utilized for any combination of sectoral tilts.

Finally, *stock selection* is a portfolio allocation strategy based on picking mispriced stocks. It uses security alphas to identify overvalued and undervalued stocks. The manager can then adjust the asset proportions in the portfolio to maximize specific return (discussed below, on page 20). These active holdings, in both positive and negative directions, increase residual risk and portfolio alpha. The primary objective of this strategy is to manage asset holdings so that any change in incremental risk is compensated by a comparable change in return. Barra MFMs facilitate stock selection by extending the risk model down to the individual equity level.

*Figure 5 on page 15* illustrates the typical prevalence of these various types of risk in a single stock, a small portfolio, and a multiple-portfolio situation. In each case, the manager’s goal is to earn a superior return with minimum risk. The use of a multiple-factor model permits the manager to pursue these active management strategies with maximum precision.

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3 A stock with a price that is low relative to its alpha is deemed undervalued.
Fixed-Income Portfolio Strategies

For fixed income portfolios, the strategies include market timing, sectoral rotation, and bond selection.

As with equities, *market timing* is the process of altering market risk exposure based on short-term forecasts in order to earn superior returns. Asset allocation is based on predictions on which way interest rates are going. The manager makes the portfolio more or less sensitive to the interest rates by altering the maturity structure. Allocating more capital to long-term holdings makes the portfolio more sensitive to interest rate changes; allocating more to short-term holdings makes it less sensitive. Demographic shifts, economic trends, and socio-political changes are closely watched to anticipate changes in the yield curve. But this strategy is much more difficult to implement with fixed income instruments than with equities. Barra MFM assists market timing by forecasting forward rate curves for any market and for any yield curve shape and by providing a detailed analysis of exposure to interest rates for any portfolio.
The second type of active management, *sectoral rotation* attempts to outperform the benchmark by making anticipatory spread bets. The manager predicts changes in market price relationships between different types of bonds and allocates funds accordingly. For example, if the spread between a retailer-issued bonds and treasury bonds is anticipated to widen because of weakening consumer demand, the manager would reduce holdings of retailer-issued bonds. On the other hand, if the spread is anticipated to tighten, the manager would accumulate more retailer-issued bonds. Barra MFMs facilitate allocation decisions by forecasting the volatility of spreads and the risk rate of credit migration.

Lastly, *bond selection* is a portfolio allocation strategy based on picking undervalued bonds. Mispriced bonds are found by credit and fundamental analyses. The manager accumulates bonds whose ratings she predicts will improve because the issuer’s fundamentals are strengthening, the market has not correctly calculated the underlying cashflow of the issue, or macroeconomic directions favor the issuer. Alternatively, the manager picks bonds that have higher yields than their peers. For example, a manager who is comparing the historical price and yield of two similar treasury bonds would select the relatively cheaper one. Barra MFMs facilitate bond selection by extending the risk model down to the individual security level.

*Figure 6 on page 17* illustrates the typical prevalence of these various types of risk in a single bond, a small portfolio, and a multiple-portfolio situation. In each case, the manager’s goal is to earn a superior return with minimum risk. The use of a multiple-factor model permits the manager to pursue these active management strategies with maximum precision.
In order to implement a desired strategy, an investment portfolio manager routinely participates in activities ranging from asset risk analysis to scenario testing to portfolio optimization. Optimization refers to the process of establishing an efficient frontier, or a set of portfolios with maximum expected net return for a given level of risk, based on defined parameter constraints. For more information about optimization, see Chapter 4, “Optimization,” starting on page 89.

In order to automate routine tasks, importing and maintaining asset-level and portfolio data is of course critical for the use of the software. Barra facilitates this process by providing wide coverage to access global data and indices from a variety of institutional investment firms. Asset selection and trades will be governed by the investment firm’s philosophy and management style and influenced by the strategies discussed above.

Case studies later in this handbook explore the risk analysis, scenario, and optimization features of Barra’s Aegis and Cosmos software.
Multiple-Factor Models

Portfolio risk and return can be decomposed along two dimensions: that which is due to factors which are prevalent throughout the market and that which is due to the idiosyncratic nature of the securities in the portfolio. A multiple-factor model is a powerful tool to shed light on these sources of risk and return within a portfolio.

Single Factor

For single-factor models, the following equation describes the excess rate of return:

\[ r_j = X_j f + u_j \]  

where

- \( r_j \) = total excess return over the risk-free rate of security \( j \)
- \( X_j \) = sensitivity of security \( j \) to the factor
- \( f \) = rate of return on the factor
- \( u_j \) = non-factor or specific return of security \( j \)
Multiple Factors

We can expand this model to include $K$ factors. The total excess return equation for a multiple-factor model becomes

$$r_j = \sum_{k=1}^{K} X_{j,k} f_k + \epsilon_j$$  \hspace{1cm} \text{Eq. 4}

where

- $X_{j,k}$ = sensitivity of security $j$ to factor $k$
- $f_k$ = return of factor $k$
- $\epsilon_j$ = non-factor or specific return of security $j$

If we unfurl Equation 4, it takes the form of Figure 5 on page 23. The asset’s return is broken out into the return due to individual factors and a portion unique to the asset and not due to the common factors. In addition, each factor’s contribution is a function of the portfolio’s exposure or weight in the factor and the return of that factor.

Exposures ($X_{i,k}$)

By observing patterns over time, common factors can be identified and exposures to these factors can be determined. An exposure represents the sensitivity of an asset to a model factor. Model factors are based around market or fundamental data which is readily available for most securities, and can be computed fairly easily at the end of any month. Factors have different possible exposure values. In the Cosmos fixed-income risk model, for example, each asset has a currency risk factor exposure of either 0 or 1: the asset will be exposed to the currency factor or not exposed. Fixed-income assets may have a swap spread factor exposure equal to the spread duration.
Because these factors are based on fundamental or market information, the Barra model’s profile of a security will respond immediately to any changes in the company’s structure or the market’s behavior. Most Barra models update security exposures on a monthly basis using the last trading day’s information to compute exposures for the coming month. In the case of interest rate exposures for fixed-income assets, the Cosmos application calculates these on the fly within the application (for a discussion of the shift, twist, and butterfly exposures, see “Term Structure Risk” on page 52).

**Factor Return ($f_k$)**

We define factor returns as pure measures of the factor’s actual performance in isolation of any other effects. It is critical that this return be an unbiased estimate of how that factor actually behaves. This enhances risk analysis, portfolio construction, and performance analysis.

Since factor returns are not readily observable, we must derive their behavior. Recall that asset exposures are computed at the end of each month. Using our multiple-factor model framework and the observed asset returns over the next month, we can solve for what the factors must have returned during the course of the month. This is done with a cross-sectional regression over the returns and exposure of assets used in model estimation — known as the *estimation universe* — for that month.

**Specific Return ($\varepsilon_j$)**

Specific return is the difference between the actual return and the predicted model return on an asset. Since the model explains whatever can be attributed to common factors, specific return is that part of the return that is not associated with common factors. The use of the term “specific” refers to that aspect of the issue that is specific to the issue and unrelated to all other issues.

*Specific risk* — referred to in Barra’s Aegis product as *asset selection risk* — represents the uncertainty in asset or portfolio return that arises from the unpredictable component of the specific return.
Decomposing Risk

Barra’s multiple-factor models (MFMs) isolate components of risk based on correlations across assets sharing similar characteristics.

Active risk has two main sources: common factors, which are prevalent throughout the market, and security specific risk, which reflects the idiosyncratic nature of each asset. Both total risk and active risk can be decomposed into common-factor and asset-selection components.

 Risk decomposition enables us to identify and isolate the contribution of each of these sources to active risk:

\[ \text{var(common factor risk)} + \text{var(active specific risk)} = \text{var(total active risk)} \]

Chapter 2, “Equity Multiple-Factor Modeling” and Chapter 3, “Fixed Income Multiple-Factor Modeling” in this Handbook describe the common factors and specific risk methodology used in Barra’s models.
Advantages of MFM

Multiple-factor models offer several advantages for security and portfolio analysis:

- A more thorough breakdown of risk and, therefore, a more complete analysis of risk exposure than other methods such as single-factor approaches.
- Because economic logic is used in their development, MFM are not limited by purely historical analysis.
- MFM are robust investment tools that can withstand outliers.
- As the economy and characteristics of individual issuers change, MFM adapt to reflect changing asset characteristics.
- MFM isolate the impact of individual factors, providing segmented analysis for better informed investment decisions.
- From an applications viewpoint, MFM are realistic, tractable, and understandable to investors.
- MFM are flexible models allowing for a wide range of investor preferences and judgment.

Of course, MFM have their limitations. They predict much, but not all, of portfolio risk. In addition, they predict risk, not return; investors must choose the investment strategies themselves.
Mathematics

In constructing a multiple-factor risk model, Barra uses historical returns to create a framework for forecasting future risk. Each month, we use a representative universe of assets to estimate returns to the common factors. The analysis begins with the decomposition of asset return into a common factor piece and a specific piece, shown mathematically in Equation 5.

**Multiple-Factor Framework for Return Decomposition**

\[
    r_i = x_{i,1}f_1 + x_{i,2}f_2 + \ldots + x_{i,k}f_k + \varepsilon_i
\]

where

- \( r_i \) = excess return of asset \( i \)
- \( x_{i,k} \) = exposures of asset \( i \) to factor \( k \)
- \( f_k \) = return to factor \( k \)
- \( \varepsilon_i \) = specific return of asset \( i \)

We can represent the monthly returns for the asset universe as a single matrix equation of \( n \) assets and \( m \) factors. Each row represents one of the assets in a portfolio or bond universe.

**Return Decomposition (Matrix Format)**

\[
    \begin{bmatrix}
        r_1 \\
        r_2 \\
        \vdots \\
        r_n
    \end{bmatrix} =
    \begin{bmatrix}
        x_{1,1} & x_{1,2} & \cdots & x_{1,m} \\
        x_{2,1} & x_{2,2} & \cdots & x_{2,m} \\
        \vdots & \vdots & \ddots & \vdots \\
        x_{n,1} & x_{n,2} & \cdots & x_{n,m}
    \end{bmatrix}
    \begin{bmatrix}
        f_1 \\
        f_2 \\
        \vdots \\
        f_m
    \end{bmatrix}
    +
    \begin{bmatrix}
        \varepsilon_1 \\
        \varepsilon_2 \\
        \vdots \\
        \varepsilon_n
    \end{bmatrix}
\]
At the end of a month, we know the return of each asset as well as its factor exposures at the beginning of the month. We estimate the factor returns via regression. The factor returns are the values that best explain the asset returns.

Calculating Risk

The covariance matrix is used in conjunction with a portfolio’s weight in each asset and the factor exposures of those assets to calculate portfolio risk. Equation 6 on page 24 is the underlying form of our risk calculations:

**Portfolio Volatility or Risk**

\[
\sigma_p = \sqrt{h_p^T (XFX^T + S) h_p}
\]

where

- \(\sigma_p\) = volatility of portfolio returns
- \(h_p\) = vector of portfolio weights for \(n\) assets: \[
\begin{bmatrix}
h_1 \\
\vdots \\
h_n
\end{bmatrix}
\]
- \(X\) = matrix of factor exposures of \(n\) assets to \(m\) factors:
  \[
  \begin{bmatrix}
x_{1,1} & x_{1,2} & \cdots & x_{1,m} \\
x_{2,1} & x_{2,2} & \cdots & x_{2,m} \\
  \vdots & \vdots & \ddots & \vdots \\
x_{n,1} & x_{n,2} & \cdots & x_{n,m}
  \end{bmatrix}
  \]
Portfolio Volatility or Risk (Continued)

\[ F = \begin{bmatrix} \text{factor returns variance-covariance matrix for } m \text{ factors:} \\
\text{Var}(f_1) & \text{Cov}(f_1, f_2) & \cdots & \text{Cov}(f_1, f_m) \\
\text{Cov}(f_2, f_1) & \text{Var}(f_2) & \cdots & \text{Cov}(f_2, f_m) \\
\vdots & \vdots & \ddots & \vdots \\
\text{Cov}(f_m, f_1) & \text{Cov}(f_m, f_2) & \cdots & \text{Var}(f_m) \end{bmatrix} \]

\[ S = \text{specific risk covariance matrix} \]

Covariances, Volatilities, and Correlations

An alternative representation of variances and covariances is volatilities and correlations. The latter representation is easier to interpret. Recall that the diagonal of the covariance matrix contains factor variances so that the factor volatility (or standard deviation) is equal to

\[ \sigma_j = \sqrt{\text{var}(f_j)} \]

The correlation between two factors is equal to

\[ \rho_{i,j} = \frac{\text{cov}(f_i, f_j)}{\sigma_i \sigma_j} \]

Note that the covariance matrix can be reconstructed from the factor volatilities and correlations.

Risk Prediction with MFMs

To estimate a portfolio’s risk, we must consider not only the security or portfolio’s exposures to the factors, but also each factor’s risk and the covariance or interaction between factors.
To calculate the variance of a portfolio, you need to calculate the covariances of all the constituent components.

Without the framework of a multiple-factor model, estimating the covariance of each asset with every other asset is computationally burdensome and subject to significant estimation errors. For example, using an estimation universe of 1,400 assets, there are 980,700 covariances and variances to calculate.

\[ V(i, j) = \text{Cov}[r(i), r(j)] \]

where

\[ V(i,j) = \text{asset covariance matrix} \]

\[ i,j = \text{individual assets} \]

\[
V = \begin{bmatrix}
V(1,1) & V(1,2) & \cdots & V(1,N) \\
V(2,1) & V(2,2) & \cdots & V(2,N) \\
V(3,1) & V(3,2) & \cdots & V(3,N) \\
\vdots & \vdots & \ddots & \vdots \\
V(N,1) & V(N,2) & \cdots & V(N,N)
\end{bmatrix}
\]

Figure 8: Asset Covariance Matrix
For N=1,400 factors, there are 980,700 covariances and variances to estimate.

A multiple-factor model simplifies these calculations dramatically. This results from replacing individual asset profiles with categories defined by common characteristics (factors). Since the asset selection risk is assumed to be uncorrelated among the assets, only the factor variances and covariances need to be calculated during model estimation.4
For example, in the U.S. Equity Model (USE3), 68 factors capture the risk characteristics of equities. This reduces the number of covariance and variance calculations to 2,346. Moreover, since there are fewer parameters to determine, they can be estimated with greater precision.

4 Barra’s models allow asset selection risk to be correlated between some assets, such as two securities from the same issuer.
Chapter 2

Equity Multiple-Factor Modeling

A Barra equity risk model is the product of a thorough and exacting model estimation process. It is an extensive, detailed process of determining the factors that describe asset returns. In this chapter we provide a brief overview of this process.
Overview

Many methods exist for forecasting a stock’s future volatility. One method is to examine its historical behavior and conclude that it will behave similarly in the future. An obvious problem with this technique is that results depend on the length of history used and the type of history used. The security might have changed over time due to mergers, acquisitions, divestitures, or other corporate actions. The past contains information that may be no longer relevant to the present. Yet, this is the approach most commonly used for measuring beta.

A more informative approach uses insights into the characteristics and behavior of the stock and market as a whole, as well as the interactions between them.

Common Factors

Common factors shared by stocks — such as country membership (and trends in that market), industry membership (and trends in that industry), recent trends in the stock’s price and the market’s returns, current fundamental ratios, and trading activity—not only help explain performance, but also anticipate future volatility. Industry membership factors may significantly impact assets within an industry group, while country membership and currency factors often dominate global portfolios.

Barra’s equity risk models capture these various components of risk and provide a multifaceted, quantitative measure of risk exposure. As described in this chapter, Barra combines fundamental and market data to create risk indices. Together with industry groups, risk indices provide a comprehensive partition of risk.
Model Estimation Flow Chart

The creation of a comprehensive equity risk model is an extensive, detailed process of determining the factors that describe asset returns. Model estimation involves a series of intricate steps that is summarized in Figure 9 below, and explained in more detail in this chapter.

**Phase I: Factor Exposures**

- **Descriptor Formulas**
- **Risk Index Formulas**
  - Fundamental and Market Data
  - Descriptors
  - Risk Indices
  - Industry Allocations

**Phase II: Factor Return Estimation**

- **Asset Returns**
  - Monthly Cross-Sectional Weighted Regression
  - Factor Loadings
  - Estimation Universe

**Phase III: Risk Estimation**

- **Factor Returns**
- **Specific Returns**
  - Specific Risk Forecast
  - Covariance Matrix

*Figure 9: Data Flow for Model Estimation Process*

Ovals denote data flow; rectangles represent manipulations of and additions to data.
Data Acquisition and Cleaning

Model estimation starts with the acquisition and standardization of data. Barra uses both market information (such as price, dividend yield, or capitalization) and fundamental data (such as earnings, sales, or assets). We pay special attention to capital restructurings and other atypical events to provide for consistent cross-period comparisons.

Market data and fundamental data for each equity risk model are gleaned, verified, and compiled from more than 100 data feeds supplied by 56 data vendors. Market information is collected daily. Fundamental company data is derived from quarterly and annual financial statements.

After data is collected, it is scrutinized for inconsistencies, such as jumps in market capitalization, missing dividends, and unexplained discrepancies between the day’s data and the previous day’s data. Special attention is paid to capital restructurings and other atypical events to provide for consistent cross-period comparisons. Information then is compared across different data sources to verify accuracy.

Our robust system of checks and our data collection infrastructure, which has been continuously refined for more than 25 years, ensure that Barra’s risk models utilize the best available data.
Risk Index Formation

Descriptor Selection and Testing

Descriptor candidates are drawn from several sources. This step involves choosing and standardizing variables that best capture the risk characteristics of the assets. To determine which descriptors partition risk in the most effective and efficient way, Barra tests the descriptors for statistical significance. Useful descriptors often significantly explain cross-sectional returns.

For some descriptors, market and fundamental information is combined. An example is the earnings-to-price ratio, which measures the relationship between the market’s valuation of a firm and the firm’s earnings.

Descriptor selection is a largely qualitative process that is subjected to rigorous quantitative testing. First, preliminary descriptors are identified. Good descriptor candidates are individually meaningful; that is, they are based on generally accepted and well-understood asset attributes. Furthermore, they divide the market into well-defined categories, providing full characterization of the portfolio’s important risk features. Barra has more than two decades of experience identifying important descriptors in equity markets worldwide. This experience informs every new model we build.

Selected descriptors must have a sound theoretical justification for inclusion in the model. They must be useful in predicting risk and based on timely, accurate, and available data. In other words, each descriptor must add value to the model. If the testing process shows that they do not add predictive power, they are rejected.
Standardization

The descriptors are normalized; that is, they are standardized with respect to the estimation universe. The normalization process involves setting random variables to a uniform scale. A constant (usually the mean) is subtracted from each number to shift all numbers uniformly. Then each number is divided by another constant (usually the standard deviation) to shift the variance.

The normalization process is summarized by the following relation:

\[
\text{normalized descriptor} = \frac{\text{raw descriptor} - \text{mean}}{\text{standard deviation}}
\]

Barra regresses asset returns against industries and descriptors, one descriptor at a time, after the normalization step.

Combining Descriptors

Risk index formulation and assignment to securities is the next step. This process involves collecting descriptors into their most meaningful combinations. Though judgment plays a major role, Barra uses a variety of techniques to evaluate different possibilities. For example, we employ statistical cluster analysis to assign descriptors to risk indices.

Descriptors for the model are selected and assigned to risk indices using proprietary formulas. Risk indices are composed of descriptors designed to capture all the relevant risk characteristics of a company.

Risk index formulation is an iterative process. After the most significant descriptors are added to the model, remaining descriptors are subjected to stricter testing. At each stage of model construction, a new descriptor is added only if it adds explanatory power to the model beyond that of industry factors and already assigned descriptors.
Industry Allocation

Along with risk index exposures, industry allocations are determined for each security. Industry allocation is defined according to what is appropriate to the local country environment. Each security is classified into an industry by its operations. Barra either uses a data vendor’s allocation scheme or creates one that categorizes the assets in the estimation universe better.

Most equity models allocate companies to single industries. For the U.S. and Japan, however, sufficient data exist to allocate to multiple industries. For these markets, industry exposures are allocated using industry segment data (such as operating earnings, assets, and sales). The model incorporates the relative importance of each variable in different industries. For example, the most important variable for oil companies would be assets; for retail store chains, sales; and for stable manufacturing companies, earnings. For any given multi-industry allocation, the weights will add up to 100%.

Multiple industry allocation provides more accurate risk prediction and better describes market conditions and company activity. Barra’s multiple-industry model captures changes in a company’s risk profile as soon as new business activity is reported to shareholders. Alternative approaches can require 60 months or more of data to recognize changes that result from market prices.
Factor Return Estimation

The previous steps have defined the exposures of each asset to the factors at the beginning of every period in the estimation window. The factor excess returns over the period are then obtained via a cross-sectional regression of asset excess returns on their associated factor exposures. (For a review of how factor return estimation works, see the section “Multiple-Factor Models” starting on page 18.) The resulting factor covariance matrix is used in the remaining model estimation steps.

Covariance Matrix Calculation

Factor returns are used to estimate covariances between factors, generating the covariance matrix used to forecast risk. The simplest way to estimate the factor covariance matrix is to compute the sample covariances among the entire set of estimated factor returns. Implicit in this process is the assumption that we are modeling a stable process and, therefore, each point in time contains equally relevant information.

A stable process implies a stable variance for a well-diversified portfolio with relatively stable exposures to the factors. However, considerable evidence shows that correlations among factor returns change. In some markets periods of high volatility are often followed by periods of high volatility. The changing correlations among factor returns and the changing volatility of market portfolios belie the stability assumption underlying a simple covariance matrix.
For certain models we relax the assumption of stability in two ways. First, in computing the covariance among the factor returns, we assign more weight to recent observations relative to observations in the distant past. Second, we estimate a model for the volatility of a market index portfolio—for example, the S&P 500 in the U.S. and the TSE1 in Japan—and scale the factor covariance matrix so that it produces the same volatility forecast for the market portfolio as the model of market volatility (described in “Scaling to Match a Market Forecast” on page 38).

Scaling the Covariance Matrix

Exponential Weighting

Suppose that we think that observations that occurred 60 months ago should receive half the weight of the current observation. Denote by $T$ the current period, and by $t$ any period in the past, $t=1,2,3,\ldots,T-1$, and let $\delta = 5^{1/60}$. If we assign a weight of $\delta^{T-1}$ to observation $t$, then an observation that occurred 60 months ago would get half the weight of the current observation, and one that occurred 120 months ago would get one-quarter the weight of the current observation. Thus, our weighting scheme would give exponentially declining weights to observations as they recede in the past.

Our choice of 60 months was arbitrary in the above example. More generally, we give an observation that is half-life months ago one-half the weight of the current observation. Then we let

$$\delta = \frac{1}{5^{\text{HALFLIFE}}}$$

Eq. 7
and assign a weight of

$$w(t) = \delta^{T-t}$$

Eq. 8

The length of the half-life controls how quickly the factor covariance matrix responds to recent changes in the market relationships between factors. Equal weighting of all observations corresponds to half-life = 1. Too short a half-life effectively throws away data at the beginning of the series. If the process is perfectly stable, this decreases the precision of the estimates. Our tests show that the best choice of half-life varies from country to country. Hence, we use different values of half-life for different single country models.

**Scaling to Match a Market Forecast**

In some markets, market volatility changes in a predictable manner. We find that returns that are large in absolute value cluster in time, or that volatility persists. Moreover, periods of above normal returns are, on average, followed by lower volatility, relative to periods of below-normal returns. Finally, we find that actual asset return distributions exhibit a higher likelihood of extreme outcomes than is predicted by a normal distribution with a constant volatility.

Barra applies variants of these systematic scalings as appropriate to its single country models.¹ Before scaling is applied to any model, a scaling model is tested and validated.² If Barra can satisfactorily fit a scaling model to the volatility of a market proxy portfolio, the model is used to scale the factor covariance matrix so that the matrix gives the same risk forecast for the market portfolio as the scaling model. Only the systematic part of the factor covariance matrix is scaled.

---

¹ Some markets, such as the emerging markets, are not scaled.
The Extended GARCH Model

Barra models use different approaches to forecast future volatility levels. The use of a GARCH submodel\(^3\) is one. The following discussion lays out the general theory of extended GARCH modeling.

Denote by \( r_t \) the market return at time \( t \), and decompose it into its expected component, \( E \left( r_t \right) \), and a surprise, \( r_t \), thus:

\[
    r_t = E \left( r_t \right) + \varepsilon_t \tag{9}
\]

The observed persistence in realized volatility indicates that the variance of the market return at \( t \), \( \text{Var}(r_m)_t \), can be modeled as:

\[
    \text{Var}(r_m)_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta \text{Var}(r_m)_{t-1} \tag{10}
\]

where

- \( (r_m)_t \) = market return at time \( t \)
- \( \varepsilon_{t-1}^2 \) = recent realized volatility
- \( \text{Var}(r_m)_{t-1} \) = recent forecast of market volatility

\(^2\) The scaling models are Generalized Auto-Regressive Conditional Heteroskedasticity (GARCH) techniques or Daily Exponentially Weighted Index Volatility (DEWIV). GARCH is applied to the USE3 model used for the case studies in Barra On Campus. The DEWIV model is not presented in this Handbook.

\(^3\) The form of the variance forecasting function distinguishes GARCH models from one another.
The coefficients $\omega$ (forecasted volatility of the market), $\alpha$ (size of recent realized volatility), and $\beta$ (size of volatility forecasts) are estimated through modeling.

This equation, which is referred to as a GARCH(1,1) model, says that current market volatility depends on recent realized volatility via $\varepsilon^2_{t-1}$, and on recent forecasts of volatility via $\text{Var}(\tilde{r}_{m,t-1})$.

If $\alpha$ and $\beta$ are positive, then this period’s volatility increases with recent realized and forecast volatility. GARCH(1,1) models have been found to fit many financial time series. Nevertheless, they fail to capture relatively higher volatility following periods of below-normal returns. We can readily extend the GARCH(1,1) model to remedy this shortcoming by modeling market volatility as:

$$\text{Var}(\tilde{r}_{m,t}) = \omega + \alpha \varepsilon^2_{t-1} + \beta \text{Var}(\tilde{r}_{m,t-1}) + \theta \varepsilon_{t-1}$$

Eq. 11

If $\theta$ is negative, then returns that are larger than expected are followed by periods of lower volatility, whereas returns that are smaller than expected are followed by higher volatility.

**Specific Risk Modeling**

During factor returns estimation, Barra separates out specific returns and forecasts *specific risk*. This portion of total risk is related solely to a particular stock and cannot be accounted for by common factors. The greater an asset’s specific risk, the larger the proportion of return attributable to idiosyncratic, rather than common, factors.

Referring to the basic factor model:

$$\tilde{r}_i = \sum_k \chi_{ik} \tilde{f}_k + \tilde{e}_k$$

Eq. 12
The specific risk of asset $i$ is the standard deviation of its specific return, $\text{Std} \left( \tilde{e}_i \right)$. The simplest way to estimate the specific risk matrix is to compute the historical variances of the specific returns. This, however, assumes that the specific return distribution is stable over time. Rather than use historical estimates, we model specific risk to capture fluctuations in the general level of specific risk and the relationship between specific risk and asset fundamental characteristics.

An asset’s specific risk is the product of the average level of specific risk that month across assets, and each asset’s specific risk relative to the average level of specific risk. Moreover, our research has shown that the relative specific risk of an asset is related to the asset’s fundamentals. Thus, developing an accurate specific risk model involves a model of the average level of specific risk across assets, and a model that relates each asset’s relative specific risk to the asset’s fundamental characteristics.

**Methodology**

Denote by $S_t$ the average level of specific risk across assets at time $t$, and by $V_{it}$ (asset $i$'s specific risk relative to the average). In equation form,

$$\text{Std} \left( \tilde{u}_{it} \right) = S_t \left( 1 + V_{it} \right)$$

Eq. 13

where

- $\text{Std} \left( \tilde{u}_{it} \right) =$ asset specific risk
- $S_t =$ average level of specific risk at time $t$
- $V_{it} =$ asset $i$'s specific risk relative to the average
We estimate a model for the *average level of specific risk* via time-series analysis, in which the average level of realized specific risk is related to its lagged values and to lagged market returns. Similarly, we estimate a model of relative specific risk by regressing realized relative specific risks of all firms, across all periods, on the firm fundamentals, which include the Barra risk factors.

For the formulae for modeling the average and relative levels of specific risk, estimating the scaling coefficients, and obtaining the final specific risk forecast, see Appendix C, “Equity Asset Selection Risk Modeling,” starting on page 31.

**Updating the Model**

Last, the model undergoes final testing and updating. Tests include comparing risk forecasts against results obtained from alternative models, and comparing *ex ante* forecasts with *ex post* realizations of beta, specific risk, and active risk. For each upcoming month’s risk forecast, Barra incorporates new information from company fundamental reports and market data, and recalculates the covariance matrix.

The latest data are collected and cleaned. Descriptor values for each company in the database are computed, along with risk index exposures and industry allocations. Next, a cross-sectional regression is run on the asset returns for the previous month. This generates factor returns which are used to update the covariance matrix. Finally, this updated information is distributed to users of Barra’s applications software.
Barra’s Aegis Portfolio Manager application enables you to decompose risk in the manner shown in Figure 10. For a brief description of each risk category, see Table 1 on page 44.

**Figure 10: Risk Decomposition for Equity Models**

▷ **Note:** For Barra’s Aegis clients who subscribe to multiple-country models, the risk composition will also include country and currency risk.
### Table 1: Equity Model Risk Categories

<table>
<thead>
<tr>
<th>Risk Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Timing</td>
<td>The risk due to exposure to the market, equal to the beta times the market risk. In variance terms, it is equal to the beta squared times market variance.</td>
</tr>
<tr>
<td>Common Factors</td>
<td></td>
</tr>
<tr>
<td>• Risk Indices (Style)</td>
<td>For a listing of risk indices and constituent descriptors used in the USE3 model, see Appendix A, “USE3 Descriptors”.</td>
</tr>
<tr>
<td>• Sectors and Industries</td>
<td>For a listing of sectors and industries used in the USE3 model, see Appendix B, “USE3 Sector and Industry Definitions”.</td>
</tr>
<tr>
<td>• Covariances</td>
<td>A statistical measure of the similarity of return outcomes between risk indices and sectors.</td>
</tr>
<tr>
<td>Asset Selection</td>
<td>Risk arising from the specific or idiosyncratic behavior of assets. Asset selection risk is independent of common factor risk categories, and is uncorrelated (or negligibly correlated) with the specific risk of other companies.</td>
</tr>
</tbody>
</table>
Chapter 3

Fixed Income
Multiple-Factor Modeling

This chapter provides an overview of the fixed-income model used by Barra's Cosmos Global Risk Manager product.
Overview

Constructing the Cosmos fixed-income risk model is something like putting a puzzle together. The pieces must fit with each other and show the entire picture of risk, as shown in Figure 11.

Figure 11: Piecing Together the Cosmos Fixed-Income Risk Model

Term structure and yield spread estimation are processes that involve asset valuation. Barra’s Cosmos risk model defines five common factor risk categories: currency, emerging market, term structure, swap spread, and credit risk. Factor exposures determine the sensitivity of portfolio return to market changes. The exposure of an asset to a risk factor is the sensitivity of its excess return to a change in the factor level. For example, the exposure of a bond to a change in the average level of interest rates is its effective duration.
The component of return not explained by the common factors can be quite volatile.\(^2\) This return is called *specific return*, and it is idiosyncratic to the issue or to the issuer.\(^3\) The risk due to variation in specific return is captured by the Cosmos *specific risk* model.

### Risk Decomposition

Barra’s approach to fixed-income risk analysis starts with a decomposition of excess return into its common factor piece and a specific piece. The Cosmos model groups common factors into *global factors*, which arise in situations where currencies are mixed or when the local currency has only a small effect on the risk of an issue, and *local factors*, which affect assets within a particular market.

In a refinement of the basic risk breakdown shown in Figure 7 on page 21, Figure 12 on page 48 shows how Cosmos individually calibrates specific risk to each local market and combines it with local market common factor risk to determine total local market risk. Specific risk is discussed later in this chapter.

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2 Fixed-income assets with lower credit quality tend to have more volatile specific return.

3 The specific risk of some corporate bonds is captured with a rating migration model that depends on the issuer, not on the issue. For more information, see “Modeling Event Risk” on page 76.
Cosmos determines a local market by currency. For example, bonds denominated in the Japanese yen may be thought of as a single market. In the yen market, the main sources of risk are changes in Japan interest rates and credit spreads. However, the structure of a local market can be more complex. For example, the European Monetary Union local market contains several sovereign governments issuing a common currency.

In each local market, the common factors are changes in interest rates and changes in spreads. The common factor local market risk of a bond is determined by the volatilities of the term structure and spread risk factors in that market, the correlations between factors, and the bond exposures to the risk factors.
Except for the euro block, each local market has three interest rate risk factors. These are the first three principal components of a key rate covariance matrix: shift, twist, and butterfly (STB). The euro market has three term structure factors for each country and three that describe average changes in rates across the euro zone. Every local market has a swap spread factor. The U.S. dollar, sterling, and euro markets have detailed credit spreads (to swap) as well. For a detailed discussion of these risk factors, see “Local Market Factors” on page 51.

Bonds in any local market may be exposed to global factors. Barra has two kinds of global risk factors: currency factors and emerging market spread factors. Both are quite volatile relative to local market risk factors and therefore constitute a significant component of portfolio risk. Barra has factors for 65 currencies and 26 emerging market issuers. For a detailed discussion of global risk factors, see “Global Factors” on page 69.

One of the most important parts of the Cosmos risk model is its factor covariance matrix (for a discussion of the mathematics, see “Calculating Risk” on page 24). This matrix contains the volatilities and correlations of the common factors. Factors fall naturally into subsets determined by the local market to which they belong or to the type of risk they describe (currency, for example). These subsets impose a block structure on the covariance matrix. Each diagonal block contains covariances of a factor subset. Off-diagonal blocks contain covariances between factors in distinct subsets Figure 13 on page 50 provides a schematic look at the block structure of the covariance matrix.

The time series of local market factor returns are used to generate factor variances and covariances in the Cosmos covariance matrix. For global factors, time series of returns are based on changes in exchange rates (for currency factors) and index spread levels (for emerging market factors). Details about time series for each common factor grouping are presented later in this chapter.

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4 A covariance matrix literally contains variances and covariances. However, the information in a covariance matrix can be equally well represented as correlations and volatilities. These latter quantities are easier to interpret.
Figure 13: Graphic View of Common Factor Blocks in Covariance Matrix
Local Market Factors

Barra’s local market factors break down into subsets corresponding to individual local markets, which are distinguished by currency. Each subset contains interest rate and spread risk factors. In Cosmos, every bond is assigned to a local market according to its principal currency. The common factor piece of a bond’s local market risk is estimated from the volatilities of the factors in its local market block, the correlations among them, and the exposures of the bond to the factors.

Most Cosmos local markets contain four factors. Three of these model risk due to a change in interest rates (see “Term Structure Risk” on page 52). The fourth is a spread risk factor based on monthly changes of the spread between the swap and sovereign curves (see “Swap Spread Risk” on page 64). The only instruments not exposed to swap spread are sovereign issues denominated in the local currency.

In addition, Cosmos has detailed sector-by-rating spread factors in the United States, the United Kingdom, and euro zone (see “Credit Spread Risk” on page 66). All credit spreads are measured to the swap curve. Credit instruments denominated in the U.S. dollar, the British pound, or the euro may be exposed to one of these credit spreads.

The euro zone presents a more complicated picture due to the presence of more than one sovereign issuer. A single set of interest rate factors is not sufficient to capture the disparate credit qualities of the EMU sovereigns. Consequently, Barra provides interest rate factors for each sovereign issuer as well as factors for changes in average euro zone rates. Thus, euro-denominated sovereign bonds from EMU member countries will be exposed to the term structure factors of the country’s local market. All other euro-denominated bonds will be exposed to the general EMU term structure factors.

5 In some markets, such as Norway, there are an insufficient number of sovereign bonds to support three interest rate factors, and fewer factors are used.
Term Structure Risk

A term structure of interest rates is a curve that maps each maturity or term to the interest rate at which money can be borrowed for that maturity. Typically, it is more expensive to borrow for longer periods of time so term structures tend to slope upward. However, this is not always the case.

Since interest rates are neither bought nor sold, a term structure is a derived quantity. Each day, Barra estimates term structures for more than 25 sovereign issuers and swap curves in 16 local markets. In most cases, including all the sovereigns, these term structures are estimated using index bond prices from the previous market day’s close.

Since both rates and rate volatilities vary by term, a fixed-income risk model must take account of the term structure of interest rates.

In this section, we examine three models of term structure risk. The first has a single factor given by average changes in rates. This model is often called a duration model because the exposure of a portfolio to the single factor is effective duration. The second is a multiple-factor key rate model whose factors are changes in rates at key maturities. Finally, we look at a shift-twist-butterfly model that is based on the way term structures actually move. This model has fewer factors than a key rate model yet has virtually the same explanatory power. We compare these models below and conclude that the shift-twist-butterfly model is best.

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6 Based on monthly changes in a yield curve estimated from the pool of actively traded EMU sovereign issues. Bonds are weighted by the GDP of the issuer. See Lisa Goldberg and Anton Honikman, “The Euro Yield Curve: Projecting the Future,” Euro (December 1998).

7 In the EMU local market, Barra uses the same swap curve for all member countries.

8 Barra calculates swap spot curves from swap par yields and LIBOR rates.
Historical Perspective: Duration Models

Experience shows that in developed markets, rates tend to move up and down roughly in parallel. This observation led early risk analysts to manage money with only a single factor: average change in interest rates. **Effective duration** is the exposure of a bond or portfolio to the average change in rates. In mathematical terms, effective duration is given as follows:

**Effective Duration**

\[
D_{\text{eff}} = -\frac{1}{P} \left( \frac{\partial P}{\partial r} \right)
\]

where

- \( D_{\text{eff}} \) = effective duration
- \( \frac{\partial P}{\partial r} \) = change in price with respect to change in level of rates

To a first approximation, portfolio return is the product of effective duration and the average change in rates,

\[
\frac{\Delta P}{P} = \Delta r D_{\text{eff}}
\]

where \( \Delta P/P \) is the bond’s return. It immediately follows that the bond return variance is approximately equal to the factor variance times the squared effective duration. In mathematical terms, this is expressed as

\[
\sigma^2 \left( \frac{\Delta P}{P} \right) = \sigma^2 (\Delta r) D_{\text{eff}}^2
\]

The disadvantage of the duration model is that it does not account for slope change and reshaping of the term structure. As we will see below (page 55), the use of a new set of factors will capture these term structure movements.
Key-Rate Model

We now move from the single-factor duration model to the opposite extreme: a key-rate model. Cosmos term structures are based on “key rates” at a distinguished set of maturities. A key-rate risk model is one whose defined factors are changes in key rates. At the core of the model is the covariance matrix of key rate changes. Since the full maturity dependence is represented, this model completely describes term structure variation.

Table 2 shows the key rate correlations and volatilities of the Australia market. A close investigation of the table immediately reveals the main shortcoming of a key rate model. Neighboring key rates are typically 90% correlated. Even long and short rates have roughly 60% correlation. This means that the key rate model has more factors than necessary.

Table 2: Monthly Correlation Matrix for the Australia Market (January 1988 to January 2001)

<table>
<thead>
<tr>
<th></th>
<th>1 yr</th>
<th>2 yrs</th>
<th>3 yrs</th>
<th>4 yrs</th>
<th>5 yrs</th>
<th>7 yrs</th>
<th>10 yrs</th>
<th>20 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>1.00</td>
<td>0.91</td>
<td>0.86</td>
<td>0.83</td>
<td>0.81</td>
<td>0.72</td>
<td>0.63</td>
<td>0.56</td>
</tr>
<tr>
<td>2 yrs</td>
<td></td>
<td>1.00</td>
<td>0.97</td>
<td>0.95</td>
<td>0.93</td>
<td>0.84</td>
<td>0.75</td>
<td>0.69</td>
</tr>
<tr>
<td>3 yrs</td>
<td></td>
<td></td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
<td>0.92</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
<td>4 yrs</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>0.99</td>
<td>0.94</td>
<td>0.88</td>
<td>0.81</td>
</tr>
<tr>
<td>5 yrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>0.97</td>
<td>0.91</td>
<td>0.84</td>
</tr>
<tr>
<td>7 yrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>0.97</td>
<td>0.89</td>
</tr>
<tr>
<td>10 yrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>0.87</td>
</tr>
<tr>
<td>20 yrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

Source: Barra
Shift-Twist-Butterfly Model

The high degree of correlation among changes in key rates is just a quantitative restatement of the intuition underlying the duration model: rates move roughly in parallel. However, there is more to term structure risk than level change.

In 1991 Barra introduced a three-factor interest rate risk model. The factors reflect the way term structures actually move: shift, where rates change roughly in parallel; twist, where the short and long end of the curves move in opposite directions, steepening or flattening; and butterfly, where the curvature of the term structure changes or flexes. Together, these three principal components capture more than 95% of interest rate variation in most developed markets. The shift-twist-butterfly model (STB) remains the model of choice for interest rate forecasts.

How do we turn these intuitive term structure movements into risk factors? At Barra, we apply a principal components analysis\(^9\) to a key rate covariance matrix estimated from a history of term structure changes. This analysis amounts to a rotation of the covariance matrix to a diagonal matrix. The new factors underlying the rotated matrix are orthogonal, weighted combinations of changes in interest rates. The diagonal elements are the in-sample variances of the factors.

The characteristic shapes of shift, twist, and butterfly persist across markets and time periods. Figure 14 displays the shift, twist, and butterfly factors in the Germany market. Like most markets, the shift factor tends to be slightly humped at the short end because short rates tend to be more volatile than long rates.

\(^9\) A principal components analysis of a covariance matrix gives rise to a set of orthonormal factors and associated volatilities that can be used to reconstruct the original matrix. For more details see, for example, Richard A. Johnson and Dean W. Wichern, *Applied Multivariate Statistical Techniques*, 5th ed., Pearson Education, 2001.
As an example of how shift, twist, and butterfly account for term structure movement, consider the change in the U.S. term structure following a Federal Reserve System attempt to stimulate the U.S. economy by decreasing interest rates. This can be seen in a comparison of the spot rate curve at the time of the decrease and one month after.
Figure 15: Curve Movement After a Decrease in Interest Rates

The key information about the Fed’s actions is conveyed by the downward movement of most of the spot rates. In other words, the underlying information about each spot rate movement is the same: rates decreased. This information is conveyed with a different magnitude, however, at each vertex.

The movement of the curve can clearly be decomposed into a large shift movement (the level of interest rates went down) and a large twist movement (the slope of the curve went up).

Like any model, the STB approach has shortcomings. The most common concern is that the STB model requires frequent re-estimation. The model depends on many choices, including the time period and weighting scheme used in the estimation of the underlying key rate matrix. How are these choices made? How often do the factors need to be updated?

There are no set answers to these questions. Fortunately, overwhelming empirical evidence shows that the factors are quite stable over time. Most reasonable methodologies for building shift, twist, and butterfly factors give rise to models that accurately forecast risk. Factors only need to be updated when the market structure changes.
In January 2001, we re-estimated the term-structure risk factors for all Cosmos markets. At that time we experimented with different weighting schemes and determined that the exponential weight had little effect on the factor shapes or on their explanatory power. Further, we discovered that in most markets, the estimation period had surprisingly little effect. Figure 16 shows the shift, twist, and butterfly factors for the U.K. market estimated in 1995 and in 2000. The shapes are very close indeed. The U.K. market is not unusual in this regard; similar results were obtained in most markets.

![Figure 16: New and Old Risk Factors for the United Kingdom](image)

To take the point further, consider Table 3. It draws a comparison between the percentage of variance explained in sample by the new factors and the percentage of variance explained by the old factors out of sample over the period 1995–2000. In most cases the difference is negligible.
The large incremental change in the percentage of variance explained by the Spain factors merits attention. In particular, the introduction of long bonds in February 1998 changed the length of the market from 15 to 30 years. This supports the conclusion that when re-estimation is required it is because of a structural change in the market.

**Shift-Twist-Butterfly Factor Returns**

Each month, Barra calculates shift-twist-butterfly factor returns in each market by a cross-sectional regression of bond excess returns onto factor exposures. The regression universe is the set of bonds that are in an established market index at the start and end of the month. Barra uses the time series of STB returns along with returns to the other factors to generate the Cosmos covariance matrix.

Table 4 on page 60 shows volatilities for a sample 3x3 sub-block generated by shift, twist, and butterfly returns in the United Kingdom. Volatilities are reported in basis points per year. The annualized numbers are obtained from the monthly time series by multiplying the monthly estimate by \( \sqrt{12} \). In a developed market, the typical shift volatility is 65–100 basis points per year (roughly 20–30 basis points per month).
By definition, the exposure of a bond or portfolio to a risk factor is the sensitivity of its return to changes in the factor level. For example, effective duration is the sensitivity of return to change in average level of rates.

As we have seen, the shift factor is roughly parallel, and for non-callable bonds, the exposure to shift will be a number comparable to effective duration. Principal component analysis determines the relative weights of the factors on each of the key maturities, but not the absolute size of the factors. After calculating factor shapes, Barra normalizes the base shift factors, as well as normalizes the twist, and butterfly factors to have the same magnitude as the shift.

Term structure factor exposures are computed by numerical differentiation. The term structure is shocked up and down by a small scalar multiple of the factor and the bond is revalued. The difference between “up” and “down” values is divided by twice the size of the shift. The mathematical formula is given as:

<table>
<thead>
<tr>
<th>Annual Volatility (basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift 67.6</td>
</tr>
<tr>
<td>Twist 35.7</td>
</tr>
<tr>
<td>Butterfly 18.0</td>
</tr>
</tbody>
</table>

Source: Barra
Many fixed-income investors are accustomed to using key rate durations (KRD) as hedging tools. Key rate durations measure the sensitivity of portfolio return to the change in the level of a particular interest rate. The relationship between the Cosmos term structure factor exposures and key rate durations is given in the next formula.

In particular, if the factor is the average change in rates, all the weights are equal to 1, and the exposure is effective duration. We thus obtain the well-known formula

\[ D_{\text{eff}} = \sum D_i \]
Spread Risk

Historically, international bond managers focused largely on government bond issues. Recently, managers have increased the exposure of their global fixed-income portfolios to corporate and agency bonds, foreign sovereigns, supranationals, and credit derivatives. These bonds are subject to spread risk.\(^\text{10}\)

Spread risk in bond portfolios arises for two reasons: market-wide spread risks and credit event risks. Market-wide spread risk arises from changes in the general spread level of a market segment. For example, the spreads of BBB-rated telecom bonds might widen. Credit event risk arises when an individual issuer suffers an event that affects it alone. It is the risk associated with changed in company fundamentals. For example, Ford Motor Company’s car sales might fall relative to other automobile makers due to a product recall and bad publicity.

In each market, a single factor accounts for changes in the difference between the swap and sovereign curves. For markets with detailed credit models (such as the United States, United Kingdom, Japan, and euro zone), Barra decomposes credit spread risk into two components that are modelled separately.

Over the period of January 1993 to September 2002, yield spreads on BBB bonds ranged from a low of 70 basis points to a high of 240 basis points. Figure 17 on page 63 shows a time series of monthly changes in BBB spreads over the U.S. treasury curve. The figure shows a spike in volatility beginning with the currency crisis in autumn of 1998 followed by a period of persistently high volatility. Risk of this type is modeled with spread factors, described in detail below.

---

\(^{10}\) The market perceives varying levels of creditworthiness among EMU sovereigns, giving rise to spreads between EMU sovereign issuers, so we estimate a term structure for each EMU sovereign.
A Layered Approach to Spreads

There is no universally accepted choice of benchmark against which to measure credit spread. In the United States, Japan, and United Kingdom, credit spreads have historically been measured against the local treasury curve, which has been interpreted as the curve of default-free rates. However, sovereign budget surpluses and other changes in monetary policy\footnote{For example, after October 2001 the U.S. government stopped issuing 30-year treasury bonds.} have corrupted treasury curves with risk premia. Further, there is no single sovereign curve that can be used to price treasuries across the euro zone. The result of these conditions is that many investors have begun to look to the swap curve as a benchmark against which to measure credit spreads.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure17.png}
\caption{Changes in BBB Credit Spreads for Bonds Denominated in U.S. Dollars.}
\end{figure}
These considerations have led Barra to take a layered approach to modeling spread risk. In each Cosmos local market, a single factor models fluctuations between the swap and sovereign curves. Further, in the United States, United Kingdom, and euro zone, a block of sector-by-rating factors capture the risk of fluctuations in credit spreads over the swap curve. In these markets, spread risk is decomposed into swap spread risk and risk due to credit spreads over the swap curve. In markets where there are not enough data to support sector-by-rating models, we account for variable credit quality by scaling the bond exposures to swap spread risk (as described in the next section). The same scaling approach also applies to U.S. dollar-, British pound-, and euro-denominated securities that are subject to default risk but cannot be mapped to one of the credit spreads.12

**Swap Spread Risk**

In each Cosmos local market, swap spread risk is based on a single factor: the monthly change in the average spread between the swap and treasury curve. Figure 18 on page 65 shows a time series of monthly changes in the Australian dollar swap spread. This series and others of the same type are fed into the Cosmos covariance matrix estimation.

---

12 For example, a rating may be missing or the issuing firm may be unrated.
For bonds that are exposed to a credit factor or an emerging market factor in addition to the swap spread factor, the swap spread exposure is equal to the sensitivity of a bond’s return to change in the swap spread level. For most bonds in this category, this exposure is equal to the effective duration.\(^{13}\)

Bonds that do not have additional factors to explain their credit risk use the swap spread factor to forecast risk. Since bonds of lower credit quality tend to be more volatile, we scale their exposure to the swap spread by their (higher) option-adjusted spread (OAS), which is typically less than 10 basis points.

For such bonds, the sensitivity to a change in the swap spread level (typically spread duration) is scaled by the ratio of the bond’s OAS to the swap spread level so that bonds with higher OAS will have correspondingly higher volatility forecasts.

\(^{13}\) For floating rate notes and mortgage-backed securities, Barra computes the exposure to swap spread differently, as the spread duration is not equal to the effective duration.
Credit Spread Risk

The Cosmos local market risk model includes credit risk factors in three of the most active markets: U.S. dollar, sterling, and euro. Empirical evidence indicates that credit risk is currency-dependent. As a result, Barra's model is based on currency-specific credit risk factors.

In each market, most factors are determined by combinations of sector and rating. The one exception is the CCC factor in the U.S. block, which does not take account of sector. Table 5 on page 67 shows the factors by market for the U.K., EMU, and U.S. Also note that the Canadian sector in the U.S. block applies to a U.S. dollar-denominated bond issued either by the Canadian government or by an issuer domiciled in Canada.

Monthly credit spread factor return series are generated by taking the average change in spreads for bonds present in a particular sector-by-rating category at both the start and end of each period.\textsuperscript{14} When fewer than six bonds are available in a category, the return for that sector-by-rating bucket is generated by all bonds with the same rating, independent of sector.

\textsuperscript{14} The average is duration weighted. The resulting factor return is equal to the spread return to a portfolio consisting of all bonds in the estimation weighted equally by value.
Table 5: Credit Spread Factors for Sectors and Ratings

<table>
<thead>
<tr>
<th>Euro</th>
<th>Sterling</th>
<th>U.S. Dollar</th>
</tr>
</thead>
</table>
| Sector by Rating: 7 x 4 factors estimated for the seven sectors listed below and four rating categories, AAA to BBB  
  • Agency  
  • Financial  
  • Industrial  
  • Pfandbrief  
  • Sovereign  
  • Supranational  
  • Utility | Sector by Rating: 4 x 4 factors estimated for the four sectors listed below and four rating categories, AAA to BBB  
  • Financial  
  • Industrial  
  • Sovereign  
  • Supranational | Agency: One factor for all agency bonds  
  Sector by Rating: Up to 9 x 6 factors estimated for the nine sectors listed below and six rating categories, AAA to B  
  • Canadian-issued bond (any sector)  
  • Energy (industrial sector–oil and gas subsector)  
  • Financial  
  • Industrial (all industrial subsectors other than aerospace and airlines, oil and gas, railway, or shipping)  
  • Supranational (supranational issuers only)  
  • Telecommunications (utility sector–telephone subsector)  
  • Transportation (industrial sector–aerospace and airlines, railways, and shipping subsectors)  
  • Utility (utility subsectors other than telephone)  
  • Yankee (non-U.S., non-Canada, non-supranational issuer)  
  CCC Rating: One factor estimated across all sectors |

Figure 19 on page 68 shows a comparison of volatility estimates for factors common to the U.S. dollar, sterling, and euro blocks. U.S. dollar volatilities are higher than euro volatilities, sometimes by a factor of three. The volatility of the sterling sectors consistently lies between the two extremes. This is part of the motivation to use currency-dependent factors.

Figure 19 also shows a comparison of term structure shift, swap, and several credit spread volatilities for the U.S. dollar, sterling, and euro markets.
The exposure of a credit instrument to the factor with matching currency, sector, and rating is spread duration. Mathematically, spread duration is given by the following formula:

\[ D_{spr} = -\frac{1}{P_{diry}} \left( \frac{\partial P_{diry}}{\partial S_{spr}} \right) \]

where

- \( P_{diry} \) = dirty price of bond
- \( S_{spr} \) = parallel spread shift

Figure 19: Cross-Market Volatility Comparison.

U.S. dollar factor return volatilities generally exceed euro and sterling by a factor of two to three. Estimates as of May 31, 2001, based on monthly data weighted exponentially with a 24-month half-life.
Global Factors

A bond may be exposed to a global risk factor regardless of which local market it belongs to. In Cosmos 3.0, global factors are used to model two types of risk: currency risk and risk due to changes in emerging market spreads.

Currency factors must be global since currency risk is a matter of perspective. Consider a Norway-based investor with a portfolio of U.S. treasuries. U.S. treasuries belong to the U.S. dollar local market and their interest rate risk comes from changes in U.S. treasury rates. However, our Norwegian investor is also subject to the risk that comes from changes in the U.S. dollar/Norwegian kroner exchange rate.

It may be somewhat surprising to find that emerging market spread risk is handled through global factors since corporate spread risk is part of local market risk. The reason for this is economic and can be seen at a glance in Figure 21 on page 75. Emerging market risk can be enormous compared to other sources of risk and is largely dictated by the level of distress of the emerging market. Hence, the currency in which a security is denominated will not have a material effect on the risk forecast.

Because currency and emerging market risk are time sensitive, variances and covariances of global factors are estimated separately using high-frequency data and models with short memories. In addition, Barra estimates correlations between global and local market factors. These correlations are derived from monthly data weighted exponentially with a 24-month half-life.

---

15 In a distressed market, the risk forecast can exceed 2000 basis points per year.

16 The Cosmos risk model forecasts risk for a monthly horizon using monthly changes in level as the basis for local market factor volatility and correlation forecasts. Covariances between factors estimated from data histories with different frequencies will necessarily be constrained by the coarser time resolution.
Barra’s risk model combines the local market covariance matrix and global risk blocks with the correlations between local market and global factors to form the common factor risk matrix.

Currency Risk

The fluctuation of currency exchange rates is a significant source of risk faced by international investors. Often more than half the variation of a well-diversified portfolio of global equities or investment-grade bonds can be attributed to currencies. Consequently, for an international investor making currency bets or trying to hedge currency risk entirely, it is essential to have an accurate, reliable risk forecasting model that includes currency risk.

To capture the dynamics of risk in currency markets, the Barra currency risk factor block is effectively a covariance matrix combining two models: a volatility model and a correlation model.

Currency volatility levels behave like many other economic time series, varying dramatically across currencies and over time. Changes in volatility range from gradual to abrupt. General auto-regressive conditional heteroskedasticity (GARCH) models are standard tools used to forecast risk for variable volatility time series. These models posit that, conditional on today’s information, the next period return is normal with variance that is a function of previous returns and forecasts.

The daily currency forecasts are then time-scaled to conform with other Barra risk forecasts, which are based on a monthly horizon.

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17 For related discussions, see “Currency Dependence in Global Credit Markets” and “Forecasting Currency Return Volatility,” articles by Barra Research available at www.barra.com.
Numeraires and Currency Factor Exposures

The currency matrix is estimated from a U.S. dollar perspective. However, Cosmos 3.0 can linearly transform the numeraire, or base currency, to be any of the 65 currencies covered in the model.

The exposure of a bond to a currency is 1 if the bond is denominated in that currency, otherwise it is 0. Bonds with principal in one currency and interest in another have their exposures split according to their present value weights. Derivatives have no currency of denomination but rather depend on instruments denominated in a particular currency. In this case the currency exposures are the derivative deltas.

Currency Correlations

The correlation matrix is estimated using weekly currency return data relative to the U.S. dollar, exponentially weighted with a half-life of 17 weeks. This half-life is fit with a maximum likelihood estimation.

Emerging Market Risk

Much of the debt issued by emerging market sovereigns and companies domiciled in emerging markets is denominated in U.S. dollar, sterling, euro, and Japanese yen. This debt is subject to risk due to interest rates in the market indicated by the principal currency. In addition, it is subject to risk due to the creditworthiness of the issuer. Since the credit risk tends to be much larger than the interest rate risk, the factors used to model this risk are currency independent. In other words, they are global factors.
As with all other Cosmos credit spreads, emerging market spreads are measured to the swap curve. Thus, an emerging market instrument denominated in a developed market currency is exposed to the interest rate and swap spread factors in the local market determined by its currency as well as the emerging market spread indicated by its issuer. As an example, consider a bond issued by the government of Nigeria in British sterling. The bond will be exposed to the U.K. shift, twist, butterfly, swap, and currency factors as well as the Nigeria emerging market spread.

**Emerging Market Spread Risk**

The emerging market factor block forecasts risk for bonds issued in an external currency by an emerging market sovereign or by a company domiciled in an emerging market country. The returns for risk factors are estimated from changes in stripped spreads of country subindices in the J.P. Morgan Emerging Market Bond Index (EMBI) Global\(^{18}\) composed of 26 factors, one for each market.

Several markets, including Poland, South Africa, and Korea, appear in Cosmos both as developed local markets and as emerging markets. As an example, a bond will be mapped to the Korea local market factor block if it is issued by the Korean government (or a company domiciled in Korea) in Korean won. If the same bond is issued in a currency other than the Korean won, it will be mapped to the local market determined by the currency of the issue and to the Korea emerging market spread.

---

\(^{18}\) The J.P. Morgan EMBI Global is an index of dollar-denominated government debt from 26 markets. The index includes both collateralized restructured (Brady) debt and conventional noncollateralized bonds. For more information on J.P. Morgan’s spread estimation methodology, see *Introducing the J.P. Morgan Emerging Markets Bond Index Global* (August 3, 1999), available from J.P. Morgan.
Spread Volatility and Risk Estimation

As in the case of currency factors, emerging market spreads can be quite volatile and tend to exhibit variable volatility over time. Barra estimates the emerging market block separately from other factors and relies on high-frequency data. Variance and covariance estimates are based on weekly data, weighted exponentially with an eight-week half-life. The emerging market block is time-scaled to a one-month horizon and subsequently integrated into the Cosmos risk model.

Figure 20 on page 74 shows a time series of average volatility forecasts across the various market regions. We see that realized volatility was fairly stable from June 1999 to December 2000 in Africa and Asia, declined by roughly a factor of two in Latin America (ex Ecuador), and declined by a factor of four in Eastern Europe and Russia. The eight-week half-life is considerably shorter than the time scale of volatility decrease in these markets, so the risk forecasts have only slightly lagged the changes in the markets.

Figure 21 on page 75 shows volatility in emerging markets on two dates one year apart. During this period, the Ecuadorian sucre was dollarized and macroeconomic conditions improved. The resulting large decrease in volatility is obvious. In contrast, we can see the increased volatility in Argentina, following continuing political and social pressures.

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Figure 20  Annualized Volatility Forecasts (Averaged) for Regions of the EMBI Global. The volatility of the swap spread is shown for comparison.
Emerging Market Factor Exposures

An emerging market bond issued in an external currency is exposed to the spread indicated by the issuer. The exposure is the bond spread duration.\textsuperscript{20}

Credit Spread Exposure (Spread Duration)

\[
D_{pr} = -\frac{1}{P_{dirty}} \left( \frac{\partial P_{dirty}}{\partial S_{pr}} \right)
\]

where

\[
\begin{align*}
P_{dirty} &= \text{dirty price of bond} \\
S_{pr} &= \text{parallel spread shift}
\end{align*}
\]

\textsuperscript{20} Spread duration is also the exposure of a bond to credit factors.
Specific Risk

Overview

Common factors do not completely explain asset return. Return not explained by the shift-twist-butterfly and spread factors is called specific return or asset-specific return. The risk due to the uncertainty of the specific return is called asset-specific risk. This risk tends to be quite small for government bonds and investment-grade corporates. By assumption, bond-specific returns are uncorrelated with one another as well as with common factor return. Consequently, portfolio specific risk diversifies away quickly as the number of bonds in the portfolio increases. Our general model for asset-specific risk is described below in this section.

Included under the heading of specific risk is corporate bond event risk. This is the risk that a company’s debt may be repriced due to a change in perception of the company’s business fundamentals. Credit rating agencies monitor these business fundamentals in relation to a firm’s debt payment obligations, and assign debt ratings based on their estimate of the likelihood of repayment. The details of how this model is put together are explained the next section.

Modeling Event Risk

The Cosmos event risk model is based on rating migration and covers corporate issues denominated in U.S. dollar, sterling, and euro. The ingredients to the calculation are the historical rating migration matrix and average yield spreads estimated for bonds bucketed by currency and rating.
Rating Migration Matrix

Credit rating agencies update debt ratings as a firm’s condition changes. That is, credit events can trigger rating migration. For example, concerns about the debt level of the American company, Gap Inc.,21 caused Moody’s to lower their credit rating three times during a nine-month period in 2001 and 2002.22

Credit rating information is consolidated in an annualized rating migration matrix. The matrix contains historical estimates of the likelihood of debt rating migrations over the horizon of one year. Before we include the matrix in our model, we first remove any firms that transition to “non-rated” and rebalance so that once again each column sums to 100%. Next, we eliminate mathematical inconsistencies present in the observed data.

Rating Spreads and Returns

In order to forecast the risk associated with rating migration, we need to estimate the size of the impact that a rating change has on spread. We do this by calculating the spread levels for each rating category.

---


22 Changes in business fundamentals are not always accompanied by rating migration. For example, in March 2002, a money manager in a large investment company questioned the high level of short-term debt held by General Electric, a highly rated company in the United States. Spreads of GE Capital’s debt increased 8 to 15 basis points, but their credit rating remained strong. Investigating when an event will affect debt rating is an area of further Barra research. [David Feldheim, “GECC’S Credit Standing Seen Undented, Despite Criticism,” Morningstar–Dow Jones, March 22, 2002, news.morningstar.com (March 26, 2002).]
On each analysis date, bonds are grouped by rating and currency. The average spread over swap of each group is calculated. Figure 22 shows a time series of rating spread levels for U.S. dollar-denominated bonds. Note that the average spread for a bond below investment grade dwarfs the investment-grade spreads. Figure 23 on page 79 magnifies the investment-grade spreads to show more clearly their levels and relationships.

![Figure 22: U.S. Dollar Rating Spreads to Swap](image-url)
Table 6 shows a sample of U.S. dollar rating spread levels. As expected, spreads increase as credit quality diminishes.

Table 6: U.S. Dollar Rating Spread Levels, March 31, 2001 (in basis points)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Spread Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>23.4</td>
</tr>
<tr>
<td>AA</td>
<td>45.02</td>
</tr>
<tr>
<td>A</td>
<td>79.43</td>
</tr>
<tr>
<td>BBB</td>
<td>146.09</td>
</tr>
<tr>
<td>BB</td>
<td>313.52</td>
</tr>
<tr>
<td>B</td>
<td>687.69</td>
</tr>
<tr>
<td>CCC</td>
<td>1919.43</td>
</tr>
</tbody>
</table>
For each initial and final rating pair, the spread change is the difference between the spread of the final level and the spread of the initial level.\(^\text{23}\) Note that the spread changes are zero when the initial and final ratings agree and they are negative if the initial rating is higher than the final rating. The returns generated by the spreads in Table 6 are displayed graphically in Figure 24.

\[\text{Figure 24: U.S. Dollar Spread Returns, March 31, 2001}\]

**Issuer Migration Risk Forecasting Model**

We now explain how to combine the rating migration probabilities and the level spread returns (as seen in Figure 24) to generate issuer migration risk forecasts. For any bond, this forecast is the standard deviation of price return due to change in spread level.

\(^{23}\) The common factor returns we have encountered earlier in this chapter were changes to a factor level over the course of a month. By contrast, the specific returns we consider here are differences in rating spread levels on a particular date.
In order to calculate this, we first note the current rating of the bond and compute the price return to the bond that arises from each possible rating migration. The formula that converts spread return to price return is given in the equation below:

**Converting Spread Returns to Price Returns**

\[ r_{price} = r_{spr} D_{spr} \]

where

- \( r_{price} \) = price return variance due to spread change
- \( r_{spr} \) = spread return
- \( D_{spr} \) = spread duration

As an example, consider a five-year duration AA bond. On March 31, 2001, the AA spread level was 45 basis points and the BBB spread level was 146 basis points. Hence the price return for this bond due to change in level to BBB is

\[ (45 - 146) \cdot 5 = -505 \text{ basis points} \]

The case when the end state is default requires special explanation. Here, the price return is simply the recovery rate. The recovery rate is uncertain and situation dependent. However, studies have shown that a typical recovery rate for secured senior debt is roughly 50%. Hence, we use this value in our model.\(^{24}\) In fact, the recovery value has only a small impact on the risk forecast for investment-grade bonds.\(^{25}\)

---

\(^{24}\) Barra is currently undertaking a study of recovery given default.

\(^{25}\) However, the recovery model may have a pronounced impact on bonds that are below investment grade.
Once we have calculated the price return for each possible end state, we compute the weighted standard deviation using the probabilities from the monthly market data. The formula for the rating migration risk forecast for a bond with initial rating $i$ and spread duration $D_{spr}$ is

**Forecasting Rating Migration Risk**

$$\sigma^2 = \sum_j w_{ij} \left( D_{spr} \left( r_{spr,i} - r_{spr,j} \right) - r_{\text{mean}} \right)^2 + w_{i,\text{def}} (R - r_{\text{mean}})^2$$

where

- $\sigma^2$ = price return variance due to spread change or default
- $w_{ij}$ = probability that rating $i$ bond will migrate to rating $j$ over next month
- $D_{spr}$ = spread duration
- $r_{spr,i} - r_{spr,j}$ = difference between average spread for rating $i$ and rating $j$ (spread return)
- $r_{\text{mean}} = \sum_j w_{ij} D_{spr} \Delta S_{ij} + w_{i,\text{def}} R$ = mean price return due to spread change or default
- $w_{i,\text{def}}$ = probability that rating $i$ bond will default over next month
- $R$ = recovery rate in default
Figure 25 below and Figure 26 on page 84 display time series of credit risk forecasts for the U.S. dollar spreads. Credit risk can be more conveniently expressed in terms of spread risk by dividing out by the spread duration. Sharp jumps in risk forecasts for all rating classes are apparent in the credit crash of mid-1998, due to the significant widening of spreads. However, forecasts have subsequently fallen back to their pre-July 1998 levels.

Table 7 on page 85 summarizes the model forecasts as of January 31, 2000, expressed as annualized standard deviation of interest rates or spreads. For the rating-based credit risk forecasts, a spread duration of five years is assumed. The first column shows average factor volatility forecasts for factors in different groups. The second column shows the specific or credit risk forecast for a single security.
An interesting observation from this table is that the classification of issuers into investment and speculative grade (BBB and above, and below BBB) neatly corresponds to the split between bonds whose common factor and credit risk are each less than their interest rate risk, and those for which they are greater. That is, the common factor and credit risks for ratings of BBB and above are all less than the 82 basis points of risk due to spot rate volatility, while those of lower-grade bonds are above this level. We can also see that interest rate risk is dominant for investment-grade bonds and credit risk is dominant for high-yield bonds.
Proxies for Missing Rating Spreads

The calculation of rating migration risk for a bond requires estimates for all the rating spread levels. However, in some markets there are not enough data to estimate spreads below investment grade. In these cases we extrapolate from investment-grade spreads using rating spreads from a market for which data exists (notably the U.S.). We have shown empirically that spreads between the two markets for a below-investment-grade rating are proportional to those for an investment-grade rating.

<table>
<thead>
<tr>
<th>Type</th>
<th>Common Factor (basis points)</th>
<th>Specific/Credit (basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury/Spot</td>
<td>82</td>
<td>8</td>
</tr>
<tr>
<td>Agency</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>AA</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>A</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>BBB</td>
<td>34</td>
<td>79</td>
</tr>
<tr>
<td>BB</td>
<td>92</td>
<td>155</td>
</tr>
<tr>
<td>B</td>
<td>164</td>
<td>287</td>
</tr>
<tr>
<td>CCC</td>
<td>296</td>
<td>582</td>
</tr>
</tbody>
</table>
Modeling Issue-Specific Risk

Underlying the model for specific risk of domestic government bonds is the assumption that the risk of spread return is constant. In units of price return, this gives

\[ \text{Monthly Specific Risk of Domestic Government Bonds} = D_{spr} \times b \]

where

\( \sigma \) = monthly specific risk for a domestic government bond

\( D_{spr} \) = spread duration

\( b \) = constant risk of spread return for a government bond

The constant \( b \) is calibrated to each market. Figure 27 on page 87 shows specific risk forecasts for five-year duration government domestic issues. The forecasts are made for a monthly horizon and reported in annualized terms.
For defaultable bonds in markets not covered by the credit migration model, Barra uses an extension of the issue-specific risk model.
Barra’s optimizer module in Cosmos Global Risk Manager and Aegis Portfolio Manager offers a powerful portfolio management tool that helps investment managers quickly construct optimal portfolios. It does this by automatically trading off return, risk, and transaction costs based on client-specified parameters. You can impose realistic policy or regulatory constraints and specify your parameters on an optimization, while accounting for transaction costs, short positions, penalties, and hedges to create portfolios that reflect your investment goals.
Optimizer Features

An optimization using Cosmos Global Risk Manager or Aegis Portfolio Manager can be useful regardless of your investment strategy. For example, you can

- Construct a portfolio to track an index, benchmark, or liability, maximizing active return for a given level of tracking error.
- Optimally weight your universe by incorporating estimates of relative attractiveness.
- Specify constraints and penalties on local market, currency, interest rate, sector, and credit rating exposures.
- Implement tilts towards factors that you expect to do well by assigning expected returns to industries, risk indices, and specific assets.
- Control holding size, turnover, number of assets, and transaction costs.
- Implement full or optimal currency hedging.
- Use scenario-constrained optimization to generate an “efficient frontier” or portfolios with different risk-return tradeoffs.
- Rebalance an existing portfolio to optimally match your risk and return profile.
How Optimization Works

Optimization creates an optimal portfolio by trading assets found in user-specified initial and universe portfolios. The objective is to maximize utility while taking into account any specified constraints.

The optimizer feature in Barra Aegis and Cosmos products calculates the optimization objective function as

\[
\text{objective function} = \text{utility} - \text{transaction costs} - \text{penalties}
\]

where \( \text{utility} = \text{return} - (\text{risk aversion} \times \text{portfolio variance}) \)

By maximizing the optimization objective function, you are trying to balance several competing objectives:

- Maximize expected returns.
- Minimize total risk or risk relative to a benchmark.
- Minimize transaction costs.
- Minimize penalties.

It is sometimes convenient to look at the objective function calculation as a tradeoff between expected net return (expected returns minus transaction costs minus penalties) and risk. The set of portfolios with maximum expected net return is known as the efficient frontier. The risk aversion parameter, which you adjust in the optimizer, characterizes your willingness to accept additional risk to achieve a higher level of expected net return. Therefore, each point on the frontier corresponds to a different risk aversion level, \( \lambda \).

Portfolio managers have different goals, therefore optimizations in Cosmos Global Risk Manager or Aegis Portfolio Manager allow managers to customize the parameters of the optimization to best meet their needs. One manager might have a target risk level for the portfolio and would like to find the corresponding maximum-return portfolio on the frontier. Another might have a target expected return level and the need to find the minimum
risk portfolio providing that target return. Another manager might choose an optimal portfolio by balancing the risk (as defined by the risk aversion parameter) and expected return components of the portfolio. If expected returns, transaction costs, or penalties are not included, the optimizer tries to minimize risk.

In addition to the goal of maximizing the objective function, the optimizer must meet any indicated constraints, such as the upper or lower bounds on the weights of assets, cash contributions or withdrawals, or transaction-type limitations.

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**Mean-Variance Optimization**

To create an optimal portfolio, portfolio sponsors strive for high expected returns ($r_p$), and low expected risk ($\sigma^2$). Typically risk and return increase together, so the goal is to achieve the best compromise.

*Mean-Variance* optimization provides the best compromise by choosing the portfolio to maximize the *utility*, defined as

$$U = r_p - \lambda \sigma^2$$

Eq. 14

where

$\lambda = \text{risk aversion parameter}$
When managing against a benchmark portfolio with excess return $r_B$, the benchmark can be removed from the managed portfolio’s returns

$$
\theta \equiv r_p - \frac{\text{cov}[r_p, r_B]}{\text{cov}[r_B, r_B]} r_B \equiv r_p - \beta_B r_B
$$

to arrive at the residual return.

The residual return’s expected value is called alpha,

$$
\alpha \equiv \bar{\theta}
$$

and its variance is the residual risk,

$$
\omega^2 \equiv \text{cov}[\theta, \theta]
$$

The value-added utility,

$$
U_A \equiv \alpha - \lambda \omega^2
$$

is suitable for optimizing performance against a benchmark portfolio.
Two common performance metrics, *Sharpe ratio* (SR) and the *information ratio* (IR), are maximized by the optimization:

\[ SR \equiv \frac{r_p}{\sigma} \]

\[ IR \equiv \frac{\alpha}{\omega} \]

**Constraints**

Cosmos Global Risk Manager and Aegis Portfolio Manager enable clients to set the following optimization constraint types:

- upper and lower bounds for asset holdings
- transaction cost constraints
- trading constraints
- those listed in Table 8 on page 95 (Cosmos) and Table 9 on page 96 (Aegis).

---

Table 8: Cosmos Optimizer Constraints

<table>
<thead>
<tr>
<th>Constraint Type and Constraints</th>
<th>Global</th>
<th>Local</th>
<th>Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Characteristics:</strong> OA Duration, Mod OA Duration, Nominal Duration, Convexity, OA Yield, OA Spread, Average Coupon, Current Yield, Vertex Weight, Vertex Contribution to Duration, Average Maturity</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td><strong>Barra Risk Measures:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Shift, Twist, Butterfly, and Swap Exposures and Weight-Exposures</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Credit Rating Exposures and Weight-Exposures</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Local Market:</strong> % Value or % Effective Value</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Market Category:</strong> Domestic, Eurobond, Foreign, Derivatives, Cash</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Sector:</strong> Government, Financial, Industrial, Utility, Supranational, Sovereign, Local Provincial, Agency, Cash, Pfandbriefe, Jumbo Pfandbriefe</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Quality Moody:</strong> Moody credit rating buckets Aaa to NR</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Quality S&amp;P:</strong> S&amp;P credit rating buckets AAA to NR</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Quality Model:</strong> Model credit rating buckets AAA to NR</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Asset Class:</strong> Bond (implicit exposure) and Cash Instruments (explicit exposure)</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Country of Issue:</strong> % Weight for each country and for Multinational; Barra Emerging Markets (Total)</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Currency:</strong> Total Currency, Implicit, Explicit</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
</tbody>
</table>
Table 9: Aegis Optimizer Constraints

<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Specific Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>Industries</td>
<td>For a list of industries used in the USE3 model, see Appendix B, “USE3 Sector and Industry Definitions”.</td>
</tr>
<tr>
<td>Risk Indices</td>
<td>Currency sensitivity, earnings variation, earnings yield, growth, leverage, momentum, non-estimated universe, size, size non-linearity, trading activity, value, volatility, yield</td>
</tr>
<tr>
<td>Sectors</td>
<td>Basic materials, consumer (cyclical), consumer (non-cyclical), consumer services, commercial services, energy, financial, health care, industrials, technology, telecommunications, transport, utility</td>
</tr>
</tbody>
</table>

Note: Additionally, user data and formulas can be used as Aegis constraints.
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