Chapters 15

Delta Hedging with Black-Scholes Model

Joel R. Barber

Department of Finance
Florida International University
Miami, FL 33199
1 Hedging Example

- A bank has sold for $300,000 a European call option on 100,000 shares of a nondividend paying stock

- $S_0 = 49, X = 50, r = 5\%, \sigma = 20\%,$

- $T = 20$ weeks, $\mu = 13\%$

- The Black-Scholes value of the option is $240,000$

- How does the bank hedge its risk?

2 Naked and Covered Positions

- Take no action – exposed to lose if stock increases

- Buy 100,000 shares today – exposed to lose if stock decreases
– Both positions expose the bank to risk

3 Stop-Loss Strategy

– Buying 100,000 shares as soon as price reaches $50

– Selling 100,000 shares as soon as price falls below $50

– This deceptively simple hedging strategy does not work well

4 Delta

– Delta (\(\Delta\)) is the rate of change of the option price with respect to the underlying

\[
\Delta = \frac{\partial c}{\partial S}
\]

where \(c\) is the Black-Scholes option price
5 Call Delta

- Call Delta = $N(d_1)$
- Always positive
- Always less than one
- Increases with the stock price

6 Delta Neutral Hedge

- set delta of portfolio equal to zero
- this requires that you take offsetting positions
— protected against a small change in the stock price

7 Delta Neutral Hedge for Call Option

— Buy $N$ shares of stock

— Sell one call

— Portfolio Value:

$$V = NS - C$$

— Set up hedge:

$$\frac{\partial V}{\partial S} = N - \frac{\partial c}{\partial S} = 0$$

evaluated at current stock price

— Therefore,

$$N = \Delta$$
– Hedge must be adjusted with as stock price changes
  
  * What should you do if $S$ increases?
  
  * What should you do if $S$ falls?

– Buy high, sell low strategy

8 Simple Example

– Suppose $c = 10$, $S = 100$, and $\Delta = .75$

– If stock price goes up by $1$, by approximately how much will the call increase?

– Answer: $.75

– If you write one call, how many shares of stock must you own so that a small change in the stock price is offset by the change in the short call?
9 Monte Carlo Simulation

- The hedge position must be frequently rebalanced
- Delta hedging a written option involves a “buy high, sell low” trading rule
- See Tables 15.2 (page 326) and 15.3 (page 327) for examples of delta hedging
- Sample path of stock price is simulated based upon assumed volatility and expected return
- Dynamic hedge is implemented over simulated path at regular intervals
- For each path the cost of the hedge is determined

- Answer: .75 shares
– If hedge is continuously updated, the cost of the hedge should equal Black-Scholes option price

– The simulation is repeated many times (say 1000) and sample statistics for hedge cost are computed: average and standard deviation

– Notice the average hedge cost is always more than Black-Scholes price

10 Delta Neutral Hedge with Puts

– Black-Scholes Put Delta:

\[ 1 - N(d_1) \]

– Follows from Put-Call Parity

– Put delta is always less than one and increases with the stock price

– Delta Neutral Hedge
* Buy $|\Delta|$ shares of stock

* Buy one put

11 Other "Greeks"

- Gamma
  
  * $\Gamma$ is the rate of change of delta ($\Delta$) with respect to the price of the underlying asset:

  $$\Gamma = \frac{\partial^2 c}{\partial S^2}$$

  * Rate of change of the slope

  * So for a call option the $\Gamma$ is positive because the slope increases with $S$

  * Gamma Addresses Delta Hedging Errors Caused By Curvature
- Vega (not a greek letter) is the rate of change of the value of a derivatives portfolio with respect to volatility

- Rho is the rate of change of the value of a derivative with respect to the interest rate

12 Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day

- Whenever the opportunity arises, they improve gamma and vega
– As portfolio becomes larger hedging becomes less expensive

13 Creation of an Option Synthetically

– If in theory an option can be hedged, it can also be synthesized

– Idea: form a portfolio of stock and risk-free bond that has same delta as option at a given stock price

– For example, a portfolio that is 50% stock and 50% risk-free bond has a delta of .5

14 Portfolio Insurance

– Motivation

* Put options are expensive
* Put option does not trade on individual portfolios that differ from indexes

* Why not buy a put to cover each stock in portfolio?

  – Idea: synthetically create an insured position with a dynamic portfolio of stocks and bonds.

  – In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically

  – This involves initially selling enough of the portfolio (or of index futures) to match the delta of a hypothetical put-insured portfolio

  – This involves a ”buy high, sell low” strategy

  – As value of portfolio increases beyond desired floor, the percentage of assets allocated to stocks increases
- As value of portfolio decreases toward desired floor, the percentage of assets in bonds increases.

- Strategy did not work well on October 19, 1987.

- Strategy will work poorly if stock price changes are large.

- Strategy is costly in a volatile, directionless market.