• Yield Curve Shape
  – upward sloping
  – flat
  – inverted

• Yield Curve Shifts
  – parallel
  – nonparallel
    * damped parallel
    * flattening
    * steepening
    * twisting
    * positive butterfly
* negative butterfly

– empirical work reveals that about 85% of variation in term structure can be explained by a parallel shift

• Term Structure Theories

– focus on predictive power of forward rates

– forward rate should always be positive

  * If forward rate were negative, no one would hold the long term bond

* example

  • \( z(0, 2) = 4\% \)

  • \( z(0, 1) = 10\% \)
Figure 1: spot and forward rates

\[ f(1, 2)/2 = \frac{1.02^2}{1.05} - 1 = -0.91\% \]

- expectations theory
  
  * expected future spot rate equals current forward rate over same dates

  * so

\[ E[z(a, b)] = f(a, b) \]
Figure 2: future spot rate and forward rate

- problems with theory
  - compounding troubles
  - since term structure is typically upward sloping, theory predicts that spot rates are typically increasing

- local expectations theory
  - define short spot rate as spot rate over shortest time interval of interest
* expected future short spot rate equals short forward rate

* solves compounding problem

* still predicts generally rising spot rates

  – liquidity (or term ) premium

* expected future short rate equals short forward rate minus a liquidity premium

* liquidity premium increases with maturity

* in other words, forward rates over estimate future spot rates

* therefore, increasing forward rates, associated with upward sloping term structure, are not necessarily predicting an increase in spot rates

  – market segmentation
Yield Curve Risk

- risk of nonparallel shift

- rate duration - sensitivity of bond to change in given spot rate

- set (or vector) rate durations

- overall duration is weighted sum of rate durations

- example
  * barbell that promises $100 and $200 in 3 and 10 years
  * flat term 6% term structure
  * suppose 3 year rate decreases by 50 bps
  * and 10 year rate increases by 50 bps
  * steepening of yield curve
* initial price:

\[
\frac{100}{1.03^6} + \frac{200}{1.03^{20}} = 83.748 + 110.74 = 194.48
\]

* Duration:

\[
\frac{83.748 \times 3 + 110.74 \times 10}{194.48} = 6.986
\]

* Rate durations:

3 and 7 years

* based upon rate durations:

\[
\Delta P = 83.748 \left( \frac{3}{1.03} \right) (.005) - 110.74 \left( \frac{10}{1.03} \right) (.005) = -4.1561
\]

* or a new price of

\[
194.48 - 4.1561 = 190.32
\]
actual new price:

\[
\frac{100}{(1 + .055/2)^6} + \frac{200}{(1 + .065/2)^{20}} = 190.47
\]

- Key Rate Durations

- each payment date has a rate duration

- idea: pick out a small set of key maturities

- for example: 3 months, 1, 2, 3, 5, 7, 10, 15, 20, 25, 20 years

- key rates are spot rates with key maturities

- key rate duration is sensitivity of a bond portfolio to a given change in a key rate

- Alternative approach

- based upon historical term structure changes, identify key term structure shifts
* define a duration measure with respect to each key shift

* advantages
  · considers interrelationships between spot rate changes
  · fewer key durations (2 or 3 rather than 11)
  · gives guidance as to what type of shifts are likely to occur
  · and what types of shifts you are protected against