Notes on Level II, Chapter 3

Joel Barber

Department of Finance
Florida International University
Miami, FL 33199
• Mortgage

• Mortgage Backed Security

• Amortization of Mortgage

  - time \( t \) interest:

    \[
    iM_{t-1}
    \]

  - time \( t \) principal payment:

    \[
    PMT - iM_{t-1}
    \]

  - time \( t \) principal:

    \[
    M_t = M_{t-1}(1 + i)
    \]

  - zero amortization (perpetuity) if

    \[
    PMT = iM
    \]
– negative amortization (graduated payment mortgage) if

\[ PMT < iM \]

– positive amortization if

\[ PMT > iM \]

– level payment mortgage:

\[ PMT = \frac{M_t}{A(i, n)} \]

where \( M \) is the principal value and \( i \) equals contract rate divided by 12

– payment equals interest plus scheduled principal payment

• Servicing Fee

– portion of contract mortgage rate

– servicing involves
* collecting monthly payments
* forwarding payment to loan owners
* maintaining records
* sending out late notices
* collecting late fees
* administrating escrow account
* furnishing tax information to borrowers

- **Prepayments**
  - payment made in excess of monthly payment
  - reduces principal outstanding
  - typically no prepayment penalty
  - most mortgages are nonassumable
- Because prepayment amount and timing of cash flows is uncertain

- Conditional Prepayment Rate (CPR)
  - SMM (single monthly mortality) is the fraction of principal prepaid in a given month
  - (1-SMM) is the monthly survival rate
  - (1-CPR) is annual survival rate
  - CPR is annualized SMM:
    \[ 1 - CPR = (1 - SMM)^{12} \]
  - Typically you are given the CPR and need to find SMM:
    \[ SMM = 1 - (1 - CPR)^{1/12} \]
  - Prepayment for month \( t \)
    \[ SMM \times (M_{t-1} - \text{Scheduled Principal Payment}) \]
－ or

\[ SMM \times (M_{t-1}(1 + i) - PMT_t) \]

－ scheduled payment will decline over time because of prepayments:

\[ PMT_t = \frac{M_t}{A(i, n - t)} \]

where \( n - t \) is number of remaining payments

－ or

\[ PMT_t = PMT^* \times \frac{M_t}{M_t^*} \]

where \( PMT^* \) and \( M_t^* \) are fixed payment and time \( t \) principal for a CPR = 0% mortgage

－ example 1:

* CPR = 6%

* \( M_{t-1} = \$290M \)

* scheduled payment = \$3M
* single monthly mortality

\[ SMM = 1 - (1 - 0.06)^{1/12} \]

\[ 0.5143\% \]

* prepayment:

\[ (287 - 3) \times 0.005143 = \$1.4606M \]

* example 2: see Excel spread sheet

• PSA (Public Securities Association ) Prepayment Benchmark

  – seasoned mortgage:

    * issued at least 30 months prior

    * 100% PSA ↔ 6% CPR

    * 200% PSA ↔ 12% CPR

  – unseasoned
100% PSA ↔ CPR = \[
\begin{cases}
\frac{6}{30}t & \text{for } t \leq 30 \\
6 & \text{for } t > 30
\end{cases}
\]

200% PSA ↔ CPR = \[
\begin{cases}
\frac{12}{30}t & \text{for } t \leq 30 \\
12 & \text{for } t > 30
\end{cases}
\]

- based upon CPR must determine SMM to project cash flow
- CPR is a benchmark – future prepayment experience may differ
- cash flow yield: IRR based upon CPR projected cash flows
average life:

\[
\frac{1}{12} \sum_{i=1}^{T} t \times \frac{\text{(projected principal payment)}}{\text{total principal}}
\]

- Factor Affecting Prepayment
  - current mortgage rates
  - historical prepayment experience
  - seasonal factors
  - economic activity
  - location
  - demographic characteristics

- Prepayment model
  - predicts prepayments for given mortgage pool based upon above factors
– dynamic model because prepayments change with underlying factors

– model is path dependent

– typically use Monte Carlo Simulation

– PSA benchmark is a static

• Contraction risk

  – caused by prepayments increasing when rates fall

  – if rates fall,

    * price compression

    * reinvestment at lower rate

• Extension risk
– caused by prepayments slowing when rates rise

• Duration

– contraction means duration will decrease when rates fall

– extension means duration will increase when rates increase

– same is true of average life

• CMOs

– sequential pay tranches

– accrual tranche

– floating and inverse floating tranche (pages 412 to 414)
idea: divide up sequential tranche into floating and inverse floating tranche

\* M \equiv \text{par value of sequential tranche}

\* \theta \equiv \text{ratio of floating rate tranche par value to } M

\* 1 - \theta \equiv \text{ratio of inverse floating tranche par value to } M

\* CR_F = a + r

\* CR_{IF} = K - Lr

\* i \equiv \text{periodic interest rate}

\* \text{problem: choose } a, K, L \text{ such that total payment to floating and inverse floating tranche equals cash flow available}

\* \text{cash flow to } M:

\[ Mi \]
* cash flow to both tranches:

\[ \theta M(a + r) + (1 - \theta)M(K - Lr) \]

* let’s choose \( K \) such that floating and inverse floating components cancel out:

\[ \theta Mr - (1 - \theta)MLr = 0 \]

* so

\[ L = \frac{\theta}{1 - \theta} \]

* now choose \( a, K \) such that sum of fixed components equals available interest:

\[ \theta Ma + (1 - \theta)MK = Mi \]

* so

\[ K = \frac{i - \theta a}{1 - \theta} \]

* this works so long as principal (scheduled and prepaid) is directed toward higher tranches
* so bottom tranche should be used

* once principal payments are directed toward floating and inverse floating tranches, payments are divided in accordance to principal values

* so at this point fixed and floating rate instruments amortize – or return principal

* what is the nature of the prepayment risk for floating and inverse floating tranches?

* text book example

  - total tranche = $96.5
  - floating tranche = $72.375
  - assume annual pay at rate of 7.5%
  - floating rate has quoted margin of 50 basis points
• this could be based on corporate floaters
  – not tied to inverse floaters via CMO

• \( \theta = \frac{72.375}{96.5} = \frac{3}{4} \)

• so

\[
L = \frac{\frac{3}{4}}{1 - \frac{3}{4}} = 3
\]

• and

\[
K = \frac{0.075 - (\frac{3}{4})(0.005)}{1 - \frac{3}{4}} = 28.5\%
\]

• same answers as in the text book
  – PAC and support tranches

• PO and IO only strip