Case Study: Calculating Efficient Risky Portfolio with Prohibition on Short Sales for Single Index Model

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• Assume the Single Index Model holds: market model residuals are uncorrelated.

• Define the Treynor performance measure for stock \( i \) as

\[
T_i = \frac{E[R_i] - R_F}{\beta_i}
\]

• Treynor is the excess return stock \( i \) contributes per unit of systematic risk.

• For convenience define the variance ratio as

\[
v_i = \frac{\beta_i^2 \sigma_m^2}{\text{var}(\varepsilon_i)}
\]

• Rules for determining stock included in optimal portfolio

  – Rank stocks from highest to lowest Treynor measure
– For each ranked stock, \( i = 1, 2, \ldots, N \), compute

\[
C_i = \frac{\sum_{j=1}^{i} v_j T_j}{\left(1 + \sum_{j=1}^{i} v_j\right)}
\]

– Hold all stocks \( i = 1, 2, \ldots, p \) for which

\[
T_i > C_i
\]

– Note: stock \( p \) is the last stock included in the portfolio and stocks \( p, \ldots, N \) are not held.

• Rules for determining optimal weights

– Define cut-off point as \( C^* = C_p \).

– Compute

\[
Z_i = \frac{\beta_i}{\text{var}(\varepsilon_i)} \left( T_i - C^* \right)
\]

for each stock included in portfolio.
- The optimal weights of risky portfolio are given by

\[ \frac{Z_i}{\sum_{i=1}^{N} Z_i} \]
● Determination of Optimal Weights: $C^* = 5.45$

\[ Z_1 = \left(\frac{1}{50}\right)(10 - 5.45) = .091 \]
\[ Z_2 = \left(\frac{1.5}{40}\right)(8 - 5.45) = .09562 \]
\[ Z_3 = \left(\frac{1}{20}\right)(7 - 5.45) = .0775 \]
\[ Z_4 = \left(\frac{2}{10}\right)(6 - 5.45) = .1100 \]
\[ Z_5 = \left(\frac{1}{40}\right)(6 - 5.45) = .01375 \]
\[ \text{Sum} = .3879 \]

● Continued

- $X_1 = .091/.3879 = .235$

- $X_2 = .09562/.3879 = .246$

- $X_3 = .0775/.3879 = .200$

- $X_4 = .110/.3879 = .284$

- $X_5 = .01375/.3879 = .035$
Problem. Assume risk-free rate = 4%. Determine

- which stocks should be held
- optimal weight on each stock in risky portfolio
- portfolio beta
- systematic portfolio variance and standard deviation
- unsystematic portfolio variance and standard deviation
- expected portfolio return
- portfolio standard deviation
- the plot of the efficient frontier (with risk-free asset)
- the fraction of your wealth you should allocate to the optimal risky portfolio to obtain a risk of 3.20%
weights for each stock and the risk-free asset to obtain a risk of 3.20%