Co-efficient of Determination

$$R^2 \equiv \frac{\text{Regression sum of variation}}{\text{Total sum of variation}}$$

$$= 1 - \frac{\text{Residual sum of variation}}{\text{Total sum of variation}}$$

$$= 1 - \frac{\hat{e}'\hat{e}}{(y-\bar{y}\cdot j)'(y-\bar{y}\cdot j)}$$

Remarks

① $0 \le R^2 \le 1$

② $R^2 \uparrow$ as $K \uparrow$

③ $$\bar{R}^2 = 1 - \frac{\hat{e}'\hat{e}/(T-K)}{(y-\bar{y}\cdot j)'(y-\bar{y}\cdot j)/(T-1)}$$

$$\Rightarrow \bar{R}^2 = R^2 - \frac{K}{T-K-1}(1-R^2)$$

AIC : ⑧ Akaike Information Criterion

$$AIC = \ln \frac{\hat{e}'\hat{e}}{T} + \frac{2K}{T}$$

↑ penalty term

Smaller AIC better.

* ① R^2 means the proportion of the total variation in y that is accounted for by variation in the regressors.

Theorem: In a multiple regression, $\bar{R}^2$ will fall (rise) when the variable x is deleted from the regression if the t-ratio associated with this variable is greater (less) than 1.

|t-ratio| > 1 corresponds to a two-tail probability of

$$(1 - 2 \times 0.3413) = (1 - 0.6826) = 0.3174$$

* ⑦ One possibility is to choose a regression specification that maximizes $\bar{R}^2$. However, $\bar{R}^2$ may not penalize loss of degrees of freedom enough.

Therefore, use min AIC criterion, perhaps.

* ③ Adding more regressors increases R^2 but parameter estimates become progressively less precise.

* ② For R^2 to be meaningful (as a measure of proportion of total variation in y explained by regressors), the regression must include a constant term.

* ④ R^2 comparisons across different regressions are meaningful only when the dependent variable is identical across these regressions.

This means, we cannot compare R^2 for a linear & a log-linear regression.

* ⑤ R^2 as model-selection criterion focuses only on within-sample fit of a model.

* ⑦ $\bar{R}^2$ may not penalize loss of degrees of freedom on adding regressor variables sufficiently.

* ⑨ Schwarz or Bayesian Information Criterion

$$BIC = \ln\left(\frac{e'e}{n}\right) + \frac{k \ln(n)}{n}$$

BIC has higher penalty than AIC for additional regressors.
∴ it results in a simpler model.

* ⑥ How does one choose the number of regressors in a model?