

No Predictable Components in G7 Stock Returns

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Abstract: We search for time-varying predictable components in monthly excess stock index returns over the risk free rates in the G7 countries. The predictable components provide an estimate of the expected excess returns. Our unobserved components model improves on Conrad and Kaul (1988) by taking into account fat tails widely documented in returns data. Statistical hypotheses tests fail to reveal any significant time-varying predictable components in excess returns for any of the countries, except Canada. Our results are in sharp contrast to Conrad and Kaul (1988), who do isolate time-varying expected returns in weekly size-weighted portfolio returns using the same methodology but in a Gaussian setting.

Key phrases: stock return predictability; unobserved components; fat tails; stable distributions

JEL Codes: C22, C53, G14

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Introduction

Conrad and Kaul (1988) extract expected stock returns from observed weekly returns on size-based portfolios using an unobserved components or state space model. Expected returns are assumed to be time-varying and predictable, and modeled as evolving as a first order autoregressive process. Stochastic shocks in both the observation and state equations are modeled as iid Gaussian. Kaul (1996) also provides an exposition of this methodology. However, it is now well known that stock returns have fat tails and are hence typically non-Gaussian (McCulloch 1996a). Ignoring any non-normalities that may be present in the data would lead to estimation inefficiencies.

The framework of Conrad and Kaul (1988) is extended by Bidarkota and McCulloch (2004) to account for this neglected feature of the data. The extension is non-trivial as it invalidates the optimality of the Kalman filter for extracting expected returns under non-Gaussian shocks to the state space model (Harvey 1989, Ch.3). Maximum likelihood estimation is still feasible, using the recursive algorithm of Sorenson and Alspach (1971). Using this framework, Bidarkota and McCulloch (2004) investigate the possible presence of predictable components in monthly value-weighted CRSP excess stock returns on NYSE, AMEX and NASDAQ stocks.

The computational intensity of estimation makes analysis of large numbers of datasets costly using this approach. Here, we report on results obtained in analyzing international data on stock returns. We work with monthly

excess returns on the stock market over the risk free rates in the G7 countries. Stock price indices used are the S&P/TSX index for Canada, the CAE 40 for France, DAX 40 for Germany, total market index for Italy, the Nikkei 225 for Japan, the FTSE 100 for the UK, and the S&P 500 composite index for the USA. The risk free rates are the T-bill rates for Canada, UK, and USA, interbank call money rates for France, Germany, and Italy, and money market rates for Japan. The sample periods differ for each country. All the data series were obtained from DataStream.¹

Predictability of stock returns, if any, is an important issue and has received much research attention (Fama 1991). Even small levels of predictability could potentially lead to large economic gains through suitable trading strategies that exploit this fact (Xu 2004). It is important for portfolio allocation (Barberis

¹ The data period runs from Feb80-Apr04 for Canada, Nov87-Jan04 for France, Feb86-Feb04 for Germany, Apr93-Feb04 for Italy, Dec93-Feb04 for Japan, Feb78-Apr04 for UK, and Feb65-Jan04 for USA. DataStream codes for stock price indices and risk free rates data respectively are as follows: TTOCOMP and CN13884 for Canada, FRCAC40 and PIBOR3M for France, DAXINDX and FIBOR3M for Germany, TOTMKIT and ITIBK3M for Italy, JAPDOWA and JPCAL3M for Japan, FTSE100 and LDNTB1M for the UK, and S&PCOMP and FRTBS3M for the US.

2000). It has implications for models of asset pricing (Cecchetti, Lam, and Mark 1990).

This paper is organized as follows. In section 2 we outline an econometric model for stock returns and discuss its estimation. Section 3 presents estimation results and discusses hypotheses tests of interest. Section 4 concludes with a summary of the key findings from our analysis.

2. An Econometric Model for Stock Returns

2.1 Model with Predictable Components in Stock Returns

Our most general unobserved components model for excess stock returns (labeled Model 1) with non-normal errors is as follows:

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim cZ_{1t}, \quad z_{1t} \sim \text{iid } S_\alpha(0,1) \quad (1a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_\eta cZ_{2t}, \quad z_{2t} \sim \text{iid } S_\alpha(0,1) \quad (1b)$$

Here, r_t is the observed excess return, x_t is an unobserved predictable component in it, and z_{1t} and z_{2t} are independent white noise processes.

A random variable X is said to have a symmetric stable distribution $S_\alpha(0,c)$, if its log-characteristic function can be expressed as:

$$\ln E \exp(iXt) = i\delta t - |ct|^\alpha. \quad (2)$$

The parameters $c > 0$ and $\delta \in (-\infty, \infty)$ are measures of scale and location, respectively, and $\alpha \in (0,2]$ is the characteristic exponent governing the tail

behavior, with a smaller value of α indicating thicker tails. The normal distribution belongs to the symmetric stable family with $\alpha = 2$, and is the only member with finite variance, equal to $2c^2$.

Stable distributions have thick tails, and hence, increase the likelihood of the occurrence of large shocks. Hence, big market crashes (and booms) are more likely in this setup than in a Gaussian world. Mandelbrot (1963) advocated the use of stable distributions for modeling fat tails. McCulloch (1996a) provides a comprehensive survey on the financial applications of these distributions.

Any time variation in the conditional mean excess returns is due to the presence of the predictable component x_t , assumed here to follow a simple AR(1) process. Our unobserved components model for excess stock returns is then a simple AR(1) process plus noise. It is related to the unobserved components mean-reverting model for stock prices due to Summers (1986).

A version of the unobserved components model given in Equations (1), incorporating time-varying volatility, is estimated by Bidarkota and McCulloch (2004). Attempts to estimate such conditionally heteroskedastic versions of the unobserved components model with G7 country data by maximum likelihood (discussed in sub-section 2.2 below) failed. Therefore, we abstract from time-varying volatility throughout this paper.

The null model with no predictable components in excess returns (Model 2) can be formulated as follows:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim \mathbf{C}z_t, \quad z_t \sim \text{iid } S_\alpha(0,1) \quad (3)$$

2.2 Estimation Issues

The non-Gaussian state space model in Equations (1) can be estimated using the general recursive filtering algorithm due to Sorenson and Alspach (1971). This algorithm provides the optimal filtering and predictive densities under any given distributions for the errors, and a formula for computing the likelihood function. The Appendix gives these formulae. The recursive equations for computing the filtering and predictive densities are given in the form of integrals, whose closed-form analytical expressions are generally intractable, except in very special cases. For instance, when both Equations (1a) and (1b) are linear and the errors normally distributed, the integrals can be evaluated analytically and the algorithm reduces to the well-known Kalman filter. When the integrals cannot be analytically evaluated, as in this paper when the errors are stably distributed, one can numerically evaluate these integrals (Bidarkota and McCulloch 2004) as is done here or alternatively using Monte Carlo integration techniques (Durbin and Koopman 2000). The stable density is evaluated using a fast numerical approximation due to McCulloch (1996b).

3. Empirical Results

3.1 Estimation Results

Estimation is done with excess returns expressed as percentages per annum. The maximum likelihood (ML) estimates of Models 1 and 2 are presented in Tables 1 and 2, respectively. All estimates reported are rounded off to the third decimal place. Estimates of the mean excess returns μ range from 1.641 percent per annum for USA to 7.282 percent per annum for Germany. Their standard errors are however quite large, as evident from the table. Estimates of the AR coefficient ϕ range from a low of 0.033 (in absolute value) for the UK to a high of 0.883 for Japan. The signal-to-noise scale ratio c_η ranges from a low of 0.011 for the UK to a high of 1.456 for Canada. Estimates of the characteristic exponent α are practically equal to 2 for Italy and Japan, indicating Gaussian behavior. For other countries, these estimates range from 1.645 for Canada to 1.879 for the UK. Other parameter estimates also similarly show some variation across countries.

Figures 1 plot estimates of the mean of the filter density, i.e. they plot $E(x_t | r_1, r_2, \dots, r_t)$, along with its estimated standard errors for all the countries. The mean of the filter density is an estimate of the (conditionally) expected excess returns. It is also an estimate of the predictable component in excess returns. From the figures, the predictable component appears to show sizeable time-variation for Canada, France, Germany and the US, but not for the other countries. Whether this time variation is statistically significant is tested in subsection 3.2 below.. The predictable component is a one-step ahead forecast of future excess returns.

3.2 Test for Predictable Components

The null hypothesis is that returns are random, apart from a non-zero mean. The model under the null hypothesis (Model 2) can be obtained by setting $\phi = 0$ in Model 1. Note that in this case, the two shocks, ε_t and η_t , are not separately identified, so we may add $c_\eta = 0$. The standard likelihood ratio (LR) test is not applicable since the scale ratio c_η is not identified.

Hansen (1992) derives a bound for the asymptotic distribution of a standardized likelihood ratio test statistic that is applicable in such situations. However, Hansen notes that, since his theory only provides a bound for the asymptotic distribution (as against the actual asymptotic distribution itself), use of his test may result in underrejection of the null and a subsequent loss of power. Furthermore, the implementation of his test is computationally very intensive in general, and more so for our particular problem. Therefore, we refrain from using his test here.

Since estimation of the alternative Model 1 in our case is computationally very intensive, we generate small sample critical values by estimating Gaussian versions of the null and alternative models for each country with data simulated from the estimated Gaussian null model for that country. Gaussian versions of Models 1 and 2 are discussed in subsection 3.3 below.

The LR test statistic for the null hypothesis of no (time-varying) predictable components in excess returns is reported in the last row of Table 1. Small sample p-values obtained from Monte Carlo simulations are reported next to the LR test statistics in parentheses. The hypothesis is not rejected even at the 0.10 significance level for any of the countries, except Canada for which the p-value is zero to the third decimal place. Thus, there is no evidence of a statistically significant predictable component in excess returns for any of the G7 countries, except Canada for which there is strong evidence of a predictable component.

3.3 Test for Normality

A Gaussian version of our unobserved components Model 1 (used in Conrad and Kaul 1988) has the following form:

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2c}z_{1t}, \quad z_{1t} \sim \text{iid } N(0,1) \quad (4a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim \sqrt{2c_\eta}z_{2t}, \quad z_{2t} \sim \text{iid } N(0,1) \quad (4b)$$

It is obtained by setting $\alpha = 2$ in Model 1 defined by Equations (1), and recognizing that the variance of a stable random variable $S_\alpha(0, c)$ with $\alpha = 2$ is $2c^2$.

Gaussian version of the model for excess stock returns with no predictable components takes the form:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2c}z_t, \quad z_t \sim \text{iid } N(0,1) \quad (5)$$

This is obtained by setting $\alpha = 2$ in Model 2 defined in Equation (3) earlier.

Maximum likelihood estimates of these two models with the excess returns data for the G7 countries are reported in Tables 3 and 4. Most parameter estimates show some differences when compared to the corresponding stable model estimates.

A test for normality can be based on the null hypothesis $\alpha = 2$. The LR test statistics are reported in Tables 1 and 2 for models with and without a predictable component for all countries. This LR test statistic has a non-standard distribution, since the null hypothesis lies on the boundary of admissible values for α , and, hence, the standard regularity conditions are not satisfied. See Andrews (2001) for recent advances on hypothesis testing under these conditions. The small-sample critical value at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from Monte Carlo simulations in McCulloch (1997). Thus, normality is easily rejected for all countries using this critical value, except for Italy and Japan, using models with and without a predictable component.

4. Conclusions

Our analysis fails to reveal a statistically significant predictable component in monthly excess stock index returns of the G7 countries, except Canada. Our approach relies on a state space model that takes into account fat tails widely documented in stock returns data. This finding differs sharply from

Conrad and Kaul (1988), who detect significant predictable components in weekly returns on size-based portfolios in a Gaussian setting.

Appendix. Sorenson-Alspach Filtering Equations

Let $y_t, t = 1, \dots, T$, be an observed time series and x_t an unobserved state variable, stochastically determining y_t . Denote $Y_t = \{y_1, \dots, y_t\}$. The recursive formulae for obtaining one-step ahead prediction and filtering densities, due to Sorenson and Alspach (1971), are as follows:

$$p(x_t | Y_{t-1}) = \int_{-\infty}^{\infty} p(x_t | x_{t-1}) p(x_{t-1} | Y_{t-1}) dx_{t-1}, \quad (\text{A1})$$

$$p(x_t | Y_t) = p(y_t | x_t) p(x_t | Y_{t-1}) / p(y_t | Y_{t-1}), \quad (\text{A2})$$

$$p(y_t | Y_{t-1}) = \int_{-\infty}^{\infty} p(y_t | x_t) p(x_t | Y_{t-1}) dx_t. \quad (\text{A3})$$

Finally, the log-likelihood function is given by:

$$\log p(y_1, \dots, y_T) = \sum_{t=1}^T \log p(y_t | Y_{t-1}). \quad (\text{A4})$$

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Table 1: Stable Model 1 Estimates

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim c_Z z_{1t}, \quad z_{1t} \sim \text{iid } S_\alpha(0,1) \quad (1a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_\eta c_Z z_{2t}, \quad z_{2t} \sim \text{iid } S_\alpha(0,1) \quad (1b)$$

All estimates are rounded off to the third decimal place. Hessian-based standard errors for the parameter estimates are reported in parentheses. LR ($\phi = c_\eta = 0$) gives the value of the likelihood ratio test statistic. It is a test for no predictable components in excess returns. Under this null, the distribution of the LR test statistic is non-standard (see section 3.2 in the text for an elaboration). P-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses. LR ($\alpha = 2$) gives the value of the likelihood ratio test statistic for the null hypothesis of normality. The small-sample critical value at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from simulations in McCulloch (1997).

Parameters	Canada	France	Germany	Italy	Japan	UK	USA
α	1.645(0.000)	1.867(0.000)	1.748(0.092)	2.000(0.000)	1.999(0.000)	1.879(0.000)	1.866(0.112)
μ	2.4905(3.043)	2.362(5.338)	7.282(5.745)	2.516(0.616)	1.907(8.023)	2.722(1.007)	1.641(2.760)
c	23.193 (1.297)	47.412(7.418)	40.712(7.858)	57.131(3.605)	56.527(3.857)	36.577(0.000)	32.250(2.314)
c_η	1.456(0.113)	0.327(0.478)	0.529(0.404)	0.012(0.001)	0.071(0.078)	0.011(0.019)	0.128 (0.161)
ϕ	0.100 (0.049)	0.346(0.487)	0.335(0.192)	-0.277(0.051)	0.883(0.165)	0.033(0.011)	0.801(0.311)
Log Likelihood	-1560.415	-1129.987	-1253.564	-726.749	-714.687	-1700.6109	-2502.990
LR($\alpha = 2$)	47.340	5.936	11.302	0.876	0.074	34.400	23.919
LR($\phi = c_\eta = 0$)	12.842(0.000)	0.478(0.643)	2.696(0.168)	0.912(0.478)	0.184(0.823)	0.600(0.939)	1.905(0.436)

Table 2: Stable Model 2 Estimates

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim cZ_t, \quad z_t \sim \text{iid } S_\alpha(0,1) \quad (3)$$

All estimates are rounded off to the third decimal place. Hessian-based standard errors for the parameter estimates are reported in parentheses. LR ($\alpha = 2$) gives the value of the likelihood ratio test statistic for the null hypothesis of normality. The small-sample critical value at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from simulations in McCulloch (1997).

Parameters	Canada	France	Germany	Italy	Japan	UK	USA
α	1.771(0.078)	1.873(0.069)	1.772(0.054)	1.999(0.000)	1.999(0.013)	1.876(0.054)	1.876(0.054)
μ	1.864(2.943)	2.303(6.061)	6.518(5.144)	1.016(6.431)	3.661(1.785)	2.596(3.227)	1.657(0.227)
c	34.011(1.818)	51.007(3.125)	49.384(3.284)	57.519(3.638)	57.151(3.661)	37.136(1.785)	33.405(1.386)
Log Likelihood	-1566.836	-1130.226	-1254.912	-727.205	-714.779	-1700.909	-2503.943
LR($\alpha = 2$)	38.882	5.694	10.608	0.000	0.000	35.434	22.726

Table 3: Gaussian Model 1 Estimates

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2}c z_{1t}, \quad z_{1t} \sim \text{iid } N(0,1) \quad (4a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim \sqrt{2}c_\eta z_{2t}, \quad z_{2t} \sim \text{iid } N(0,1) \quad (4b)$$

All estimates are rounded off to the third decimal place. Hessian-based standard errors for the parameter estimates are reported in parentheses. LR ($\phi = c_\eta = 0$) gives the value of the likelihood ratio test statistic. It is a test for no predictable components in excess returns. Under this null, the distribution of the LR test statistic is non-standard (see section 3.2 in the text for an elaboration). P-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses.

Parameters	Canada	France	Germany	Italy	Japan	UK	USA
μ	-1.863(4.095)	1.868(6.138)	0.722(6.593)	1.689(7.251)	3.020(7.374)	0.130(2.979)	0.527(2.413)
c	88.236(1111.696)	53.640(5.616)	43.030(44.331)	54.421(85.351)	57.111(3.699)	38.487(121.731)	26.377(47.018)
c_η	0.465(5.866)	0.204(0.464)	0.845(2.141)	0.342(5.127)	0.019(0.470)	1.096(6.096)	0.954(3.579)
ϕ	0.122(0.059)	0.627(0.903)	0.233(0.586)	-0.031(1.177)	-0.450(0.609)	-0.023(0.057)	0.079(0.301)
Log Likelihood	-1584.086	-1132.819	-1259.200	-727.187	-714.724	-1718.5400	-2514.964
$LR(\phi = c_\eta = 0)$	4.382(0.075)	0.507(0.632)	2.032(0.199)	0.036(0.896)	0.110(0.869)	0.172(0.792)	0.684(0.558)

Table 4: Gaussian Model 2 Estimates

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2c}z_t, \quad z_t \sim \text{iid } N(0,1) \quad (5)$$

All estimates are rounded off to the third decimal place. Hessian-based standard errors for the parameter estimates are reported in parentheses.

Parameters	Canada	France	Germany	Italy	Japan	UK	USA
μ	-1.396(3.397)	-0.359(6.349)	0.710(6.610)	1.016(7.029)	1.785(7.405)	-0.041(3.628)	0.705(2.245)
c	41.402(1.722)	55.449(2.800)	56.936(2.733)	57.519(3.638)	57.151(3.644)	42.220(1.690)	36.511(1.192)
Log Likelihood	-1586.277	-1133.073	-1260.216	-727.205	-714.779	-1718.626	-2515.306

Figure 1: Estimates of Expected Returns from Stable Model 1

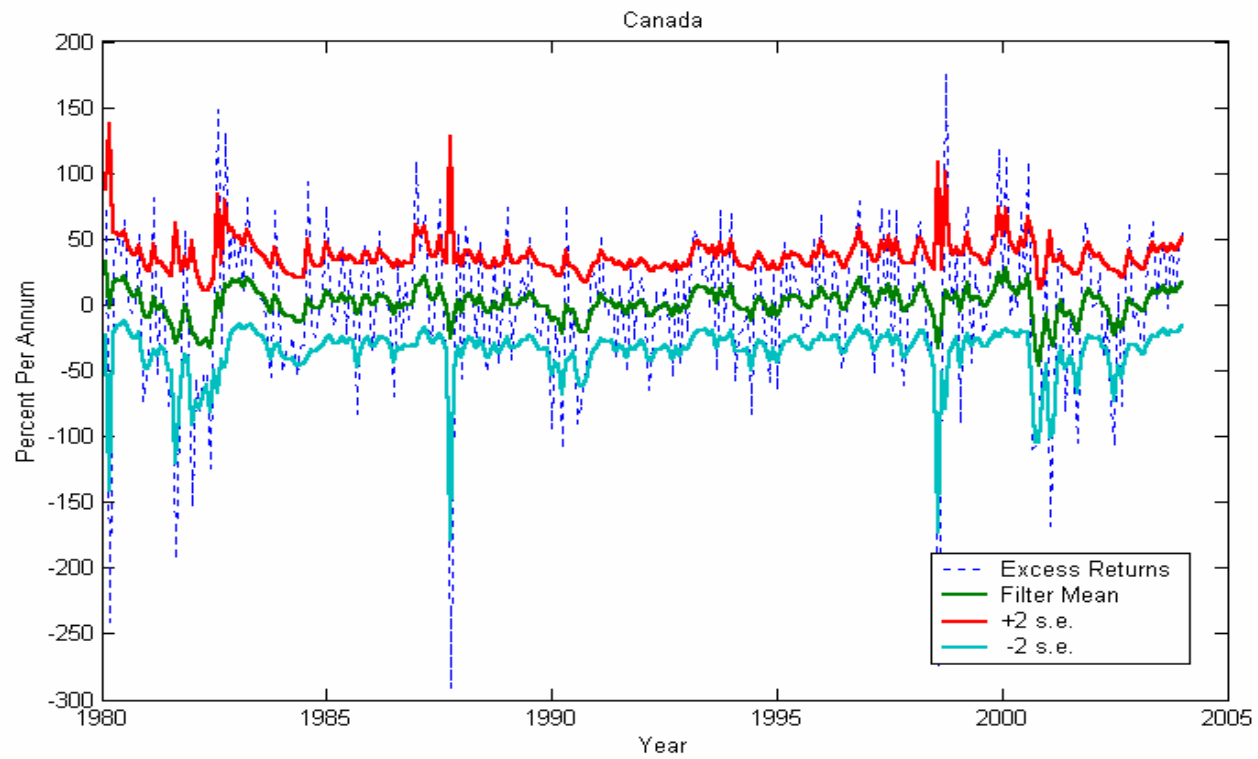


Figure 1 (contd.)

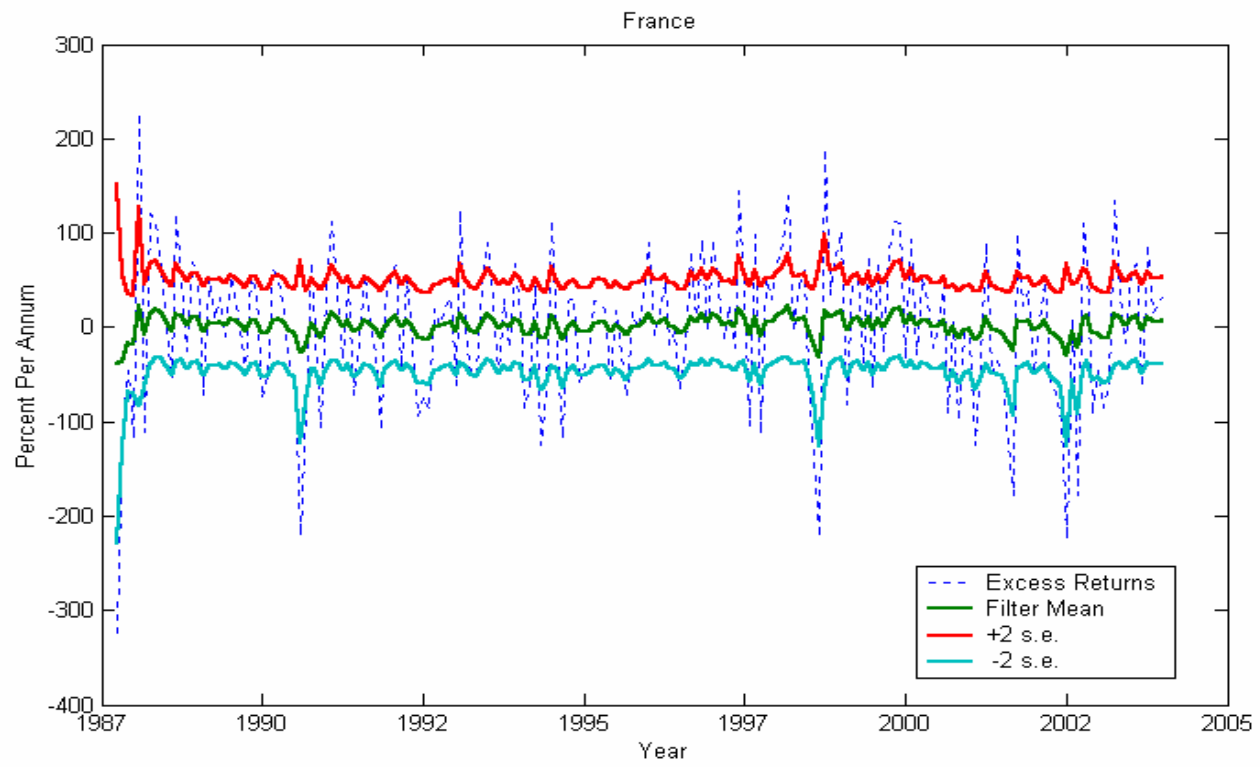


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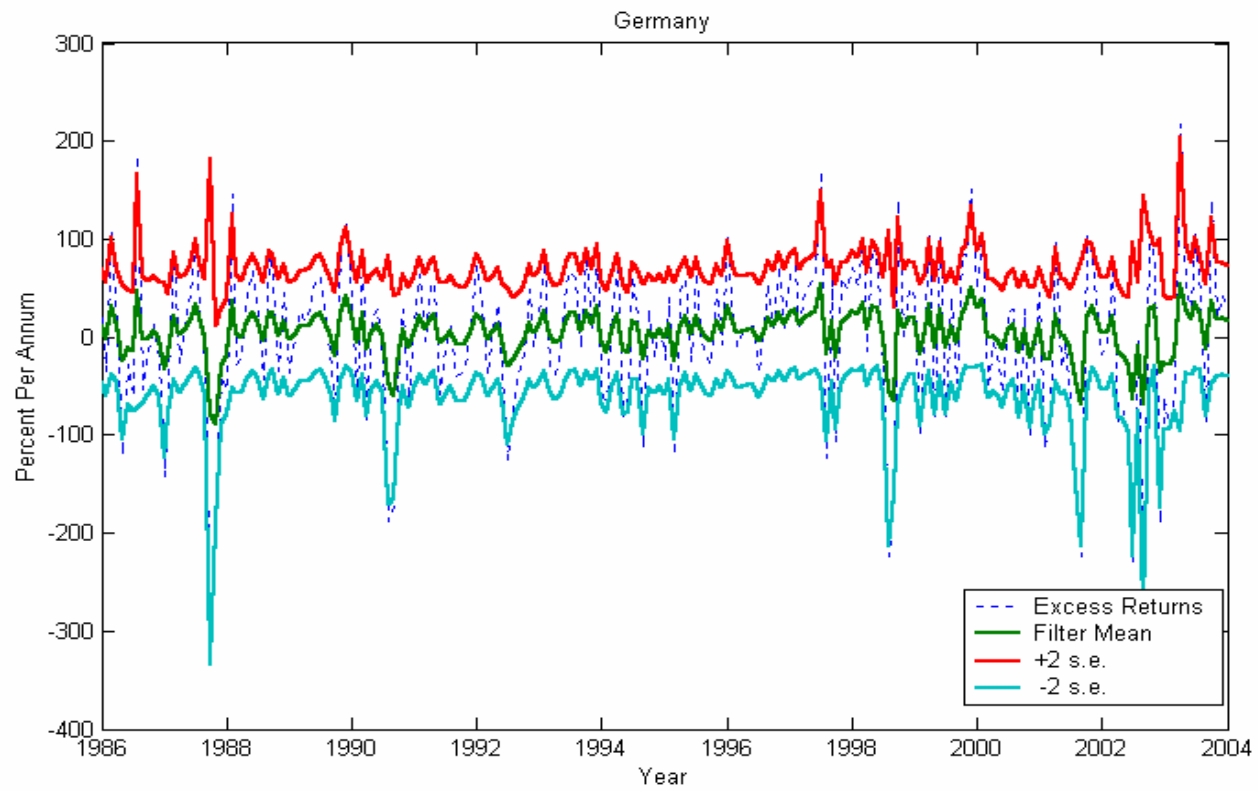


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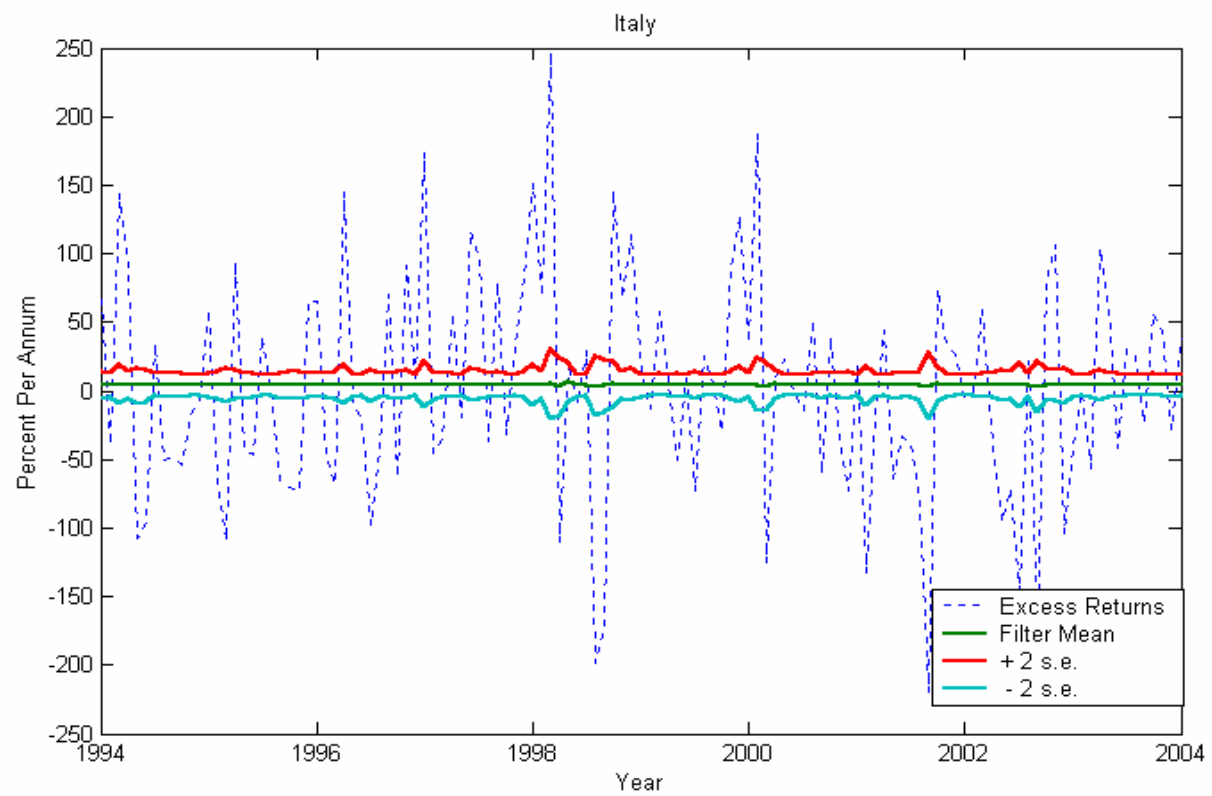


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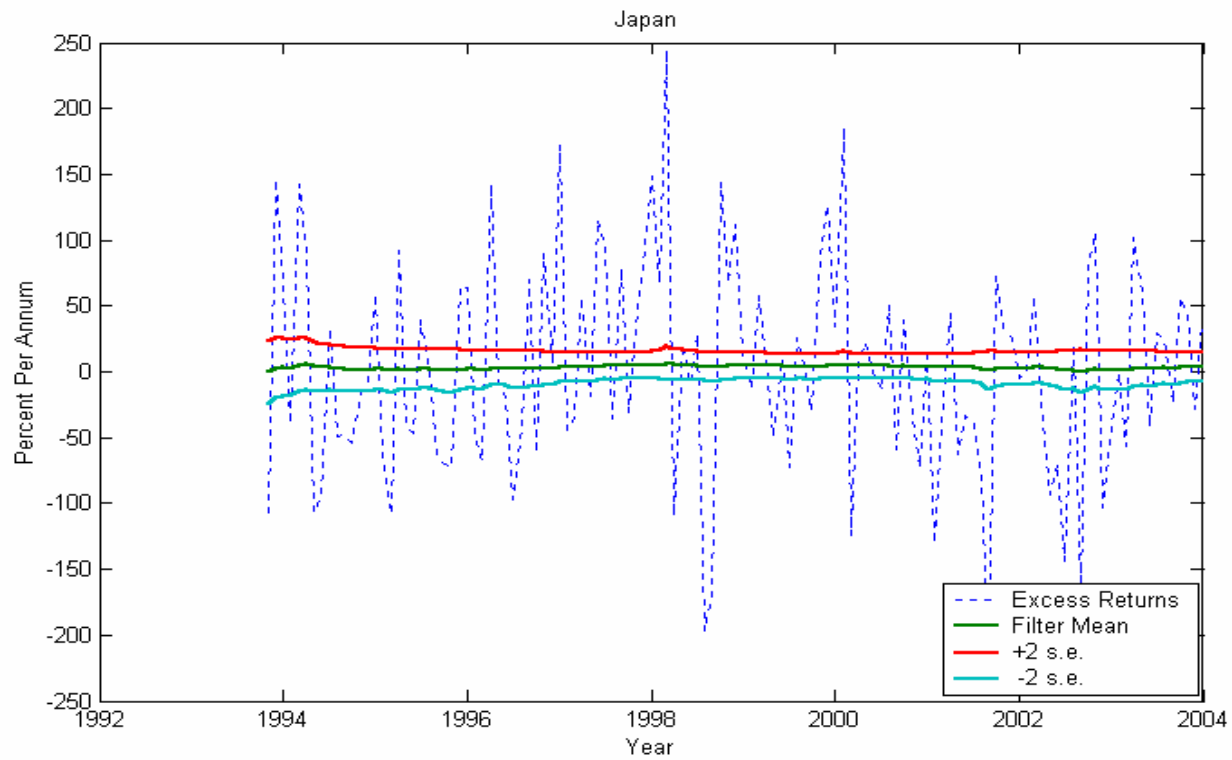


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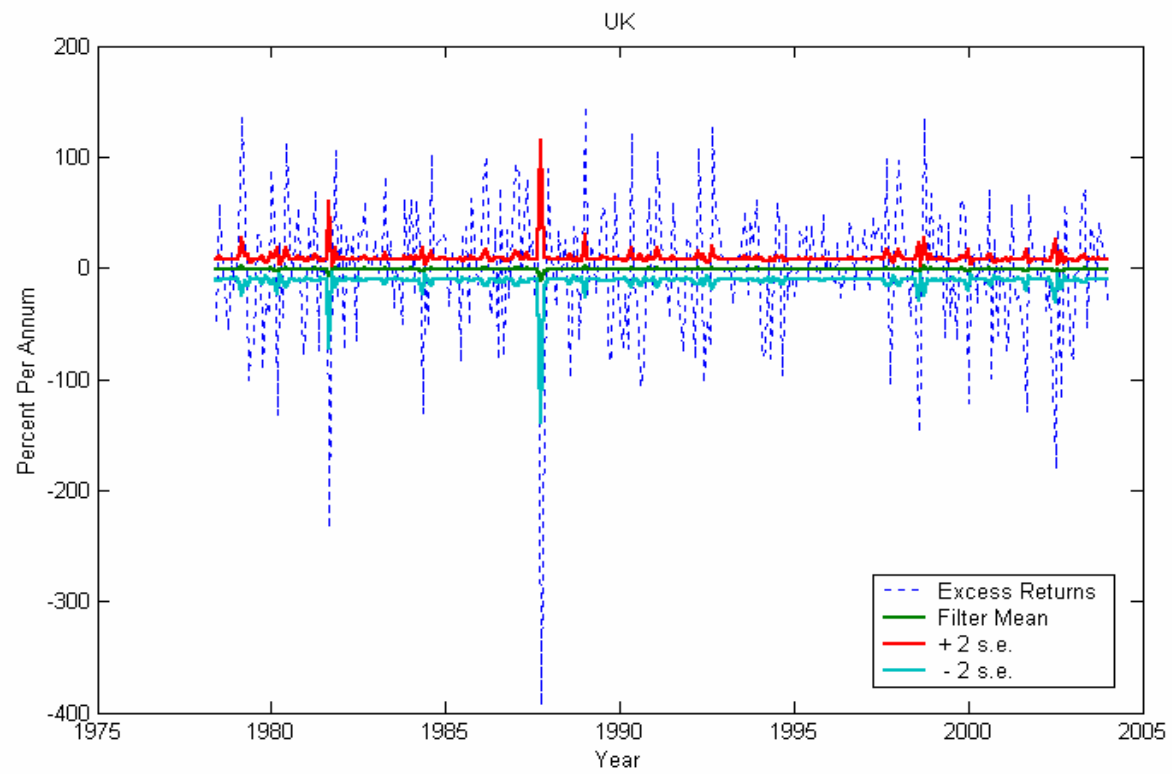


Figure 1 (contd.)

