9.8: Use the observation following Theorem 9.2 to carry out a quick calculation of the determinant of each of the following matrices:

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 4 & 2 \\
1 & 4 & 3
\end{pmatrix}
\quad \text{b) } \begin{pmatrix}
1 & 1 & 1 \\
0 & 4 & 5 \\
1 & 9 & 6
\end{pmatrix}
\]

We row reduce the first matrix by first subtracting the first row from the second and third rows, obtaining
\[
\begin{pmatrix}
1 & 1 & 1 \\
0 & 3 & 1 \\
0 & 3 & 2
\end{pmatrix}
\]
and then subtract the second row from the first, obtaining
\[
\begin{pmatrix}
1 & 1 & 1 \\
0 & 3 & 1 \\
0 & 0 & 1
\end{pmatrix}
\]
with all row-echelon matrices, we now find the determinant by multiplying the diagonal terms. Thus the matrix in (a) has determinant 3.

For the second matrix, first subtract row 1 from row 3, and then subtract twice row 2 from row 3, obtaining
\[
\begin{pmatrix}
1 & 1 & 1 \\
0 & 4 & 5 \\
0 & 0 & -5
\end{pmatrix}
\]
This has determinant \(-20\).

9.13 Use Cramer’s rule to solve the following system of equations:

\[
\begin{align*}
5x_1 + x_2 &= 3 \\
2x_1 - x_2 &= 4;
\end{align*}
\]

\[
\begin{align*}
2x_1 - 3x_2 &= 2 \\
4x_1 - 6x_2 + x_3 &= 7 \\
x_1 + 10x_2 &= 1.
\end{align*}
\]

For (a), \(\det A = -5 - 2 = -7\), and we find \(x_1 = (-3 - 4)/(-7) = 1\) and \(x_2 = (20 - 6)/(-7) = -2\).

For (b), \(\det A = -(20 + 3) = -23\), and we find \(x_1 = -23/23 = 1\), \(x_2 = 0/23 = 0\) and \(x_3 = -69/23 = 3\).

9.17 If we introduce tax rate \(t\) and let the consumption function depend on after-tax income, \(C = b(Y - tY)\), then system (10) becomes

\[
(1 - t)sY + ar = I^o + G \\
mY - hr = M_s - M^o
\]

Use Cramer’s rule to see how the equilibrium \(Y\) and \(r\) are affected by the tax rate \(t\).

Here \(\Delta = \det A = -[(1-t)sh + am]\), and then Cramer’s rule implies \(Y = -[h(I^o + G) + a(M_s - M^o)]/\Delta\)
and \(r = [(1 - t)(s(M_s - M^o) - m(I^o + G))/\Delta\). Thus

\[
Y = \frac{h(I^o + G) + a(M_s - M^o)}{(1-t)sh + am} \\
r = \frac{m(I^o + G) - (1 - t)(s(M_s - M^o) - m(I^o + G))}{(1-t)sh + am}
\]

We assume \(h(I^o + G) + a(M_s - M^o) \geq 0\) and \(m(I^o + G) - (1 - t)(s(M_s - M^o) \geq 0\), so that our answer is economically sensible. Now compute

\[
\frac{\partial Y}{\partial t} = sh\frac{h(I^o + G) + a(M_s - M^o)}{[(1-t)sh + am]^2} > 0
\]
and

\[
\frac{\partial r}{\partial t} = \frac{s(M_s - M^o)}{(1-t)sh + am} + sh\frac{m(I^o + G) - (1-t)s(M_s - M^o)}{[(1-t)sh + am]^2} \\
= sm\frac{b(I^o + G) + a(M_s - M^o)}{[(1-t)sh + am]^2} > 0.
\]

Both $Y$ and $r$ are increasing in $t$.

10.12 For each of the following pairs of vectors, first determine whether the angle between them is acute, obtuse, or right and then calculate this angle:

- a) $u = (1, 0), \hspace{1cm} v = (2, 2)$; \\
- b) $u = (4, 1), \hspace{1cm} v = (2, -8)$; \\
- c) $u = (1, 1, 0), \hspace{1cm} v = (1, 2, 1)$; \\
- d) $u = (1, -1, 0), \hspace{1cm} v = (1, 2, 1)$; \\
- e) $u = (1, 0, 0, 0, 0), \hspace{1cm} v = (1, 1, 1, 1, 1)$.

In each case, $\cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$. For (a), $\cos \theta = \frac{2}{\sqrt{2}} = 1\sqrt{2}$, so the angle is $\theta = 45^\circ$ (acute).

For (b) $\cos \theta = 0$, so the angle is $\theta = 90^\circ$ (right). For (c), $\cos \theta = \frac{3}{\sqrt{2}}\sqrt{6} = \sqrt{3}/2$, so the angle is $\theta = 30^\circ$ (acute). For (d), $\cos \theta = -\frac{1}{\sqrt{2}}\sqrt{6} = -1/\sqrt{3}$, so the angle is $\theta = 106.8^\circ$ (obtuse). For (e), $\cos \theta = 1/\sqrt{5}$, so the angle is $\theta = 63.4^\circ$ (acute).

10.15 Prove that $\|u - v\|^2 = \|u\|^2 - 2u \cdot v + \|v\|^2$.

$\|u - v\|^2 = (u - v) \cdot (u - v) = u \cdot u - u \cdot v - v \cdot u + v \cdot v = \|u\|^2 - 2u \cdot v + \|v\|^2$, where we use the fact that $u \cdot v = v \cdot u$. 