In this problem we separate the cash flows to 4 sub-annuities: A1 (A=$10, n=9, k=5\%),
A2 (A=$10, n=5, k=6\%), A3 (A=$20, n=2, k=6\%) and A4 (A=$20, n=10, k=10). The
PV of cash flows is the sum of the PV of all 4 sub-annuities. Note that A1 is an annuity
due because there is one payment at time 0.

\[ PV_0 = 10 \left( 1.05 \right)^{9} \frac{1 - (1.05)^{-9}}{0.05} + 10 \left( 1.06 \right)^{5} \frac{1 - (1.06)^{-5}}{0.06} \left( 1.05 \right)^{-8} + 20 \left( 1.06 \right)^{2} \frac{1 - (1.06)^{-2}}{0.06} \left( 1.05 \right)^{-8} \left( 1.06 \right)^{-5} \]

\[ + 20 \left( 1.10 \right)^{10} \frac{1 - (1.10)^{-10}}{0.10} \left( 1.05 \right)^{-8} \left( 1.06 \right)^{-7} \]

\[ = 177.00662 \]

\[ FV_{25} = 10 \left( 1.05 \right)^{9} \left( 1.10 \right)^{10} + 10 \left( 1.06 \right)^{2} \left( 1.10 \right)^{10} \frac{1 - (1.06)^{-5}}{0.06} \left( 1.06 \right)^{2} \left( 1.10 \right)^{10} \]

\[ + 20 \left( 1.06 \right)^{2} \frac{1 - (1.06)^{-2}}{0.06} \left( 1.10 \right)^{10} + 20 \left( 1.10 \right)^{10} \frac{1 - (1.10)^{-10}}{0.10} \]

\[ = 1,019.9334 \]
(b) 

\[ PV_0 = 10(1.08) \left[ \frac{1 - (1.08)^{-8}}{0.08} \right] + 10 \left[ \frac{1 - (1.10)^{-6}}{0.10} \right] (1.08)^{-7} + 20 \left[ \frac{1 - (1.10)^{-12}}{0.10} \right] (1.08)^{-7} (1.10)^{-6} \]

\[ = $132.36 \]

\[ FV_{25} = 10 \left[ \frac{(1.08)^8 - 1}{0.08} \right] (1.10)^{18} + 10 \left[ \frac{(1.10)^6 - 1}{0.10} \right] (1.10)^{12} + 20 \left[ \frac{(1.10)^{12} - 1}{0.10} \right] \]

\[ = $1,261.22 \]

To check, \ 132.36 = 1261.22(1.08)^{-7}(1.1)^{-18}

(c) 

\[ FV_{25} = 10(9) \left[ 1 + \frac{(9-1)(.05)}{2} + (7)(.06) + (10)(.1) \right] + 10(5) \left[ 1 + \frac{(5-1)(.06)}{2} + (2)(.06) + (10)(.1) \right] \]

\[ + 20(2) \left[ 1 + \frac{(2-1)(.06)}{2} + (10)(0.1) \right] + 20(10) \left[ 1 + \frac{(10-1)(.10)}{2} \right] \]

\[ = $235.8 + 112 + 81.2 + 290 = $719.0 \]

(d) 

\[ FV_{25} = 10(8) \left[ 1 + \frac{(8-1)(.08)}{2} + (18)(.1) \right] + 10(6) \left[ 1 + \frac{(6-1)(.10)}{2} + (12)(.1) \right] \]

\[ + 20(12) \left[ 1 + \frac{(12-1)(.10)}{2} \right] \]

\[ = $246.4 + 147 + 372 = $765.4 \]
2.  \( N = 30 \)
   Time Period = one month
   Monthly rate = 0.1/12 = 0.008333

   \[ PV(\text{Miami Auto's}) = 1,000 + 300 \left( \frac{1 - (1.008333)^{-30}}{0.008333} \right) = 8,934.5 \]

   \[ PV(\text{Kendall Auto's}) = 9,000 \]
   Therefore, Miami Auto’s provides a better deal.

3.  \( N = 30 \)
   Time Period = one year
   Periodic rate = APY or \( k \)
   \( a \)

   (a)  \( k = 12\% \)
   \[ FVA = 60,000 = A \cdot \left( \frac{(1.12)^{18} - 1}{0.12} \right), \quad A = 1,076.24 \]

   (b)  When interest is compounded monthly, \( k = \left( 1 + \frac{0.12}{12} \right)^{12} - 1 = 12.6825\% \),
   \[ $60,000 = A \cdot \left( \frac{(1.126825)^{18} - 1}{0.126825} \right), \quad A = 1,004.33, \]

   (c)  If deposits are in the beginning of the year,
   \[ FVAD = 60,000 = A \cdot \left( \frac{(1.126825)^{18} - 1}{0.126825} \right)(1.12), \quad A = 960.93 \]
   \[ FVAD = 60,000 = A \cdot \left( \frac{(1.126825)^{18} - 1}{0.126825} \right)(1.126825), \quad A = 891.31 \]

4.  (a)  \( k = 10\% \) and annual deposit:
   (i)  \( N = 18 \)
   (ii) Time Period = one year
   (iii) Periodic Rate = \( k = 10\% \)
   FV of upfront deposit: \( $5,000(1.1)^{18} = 27,799.59 \)
   Required future amount = \( $100,000 - 27,799.59 = 72,200.41 \)
   \[ $72,200.41 = A \cdot \left( \frac{(1.10)^{18} - 1}{0.10} \right), \quad A = 1,583.36 \text{ per year}. \]

   (b)  \( k_{12} = 10\% \) and monthly deposit:
   (i)  \( N = 18 \times 12 = 216 \)
   (ii) Time Period = one month
   (iii) Periodic Rate = \( k_{12}/12 = 0.008333 \)
FV of upfront deposit: $5,000(1 + .1/12)^{12 \times 18} = $30,023.47  
Required future amount = $100,000 – 30,023.47 = $69,976.53  
$69,976.53 = A \cdot \left[ \frac{(1.008333)^{216} - 1}{0.008333} \right], A = $116.52 per month.

(c) \( k_{12} = 10\% \) and annually deposit: 
(i) \( N = 18 \)  
(ii) Time Period = one year  
(iii) Periodic Rate = \( k_a = (1 + .1/12)^{12} - 1 = 10.4713\% \)  
FV of upfront deposit: $5,000(1 + .1/12)^{12 \times 18} = $30,023.47  
Required future amount = $100,000 – 30,023.47 = $69,976.53  
$69,976.53 = A \cdot \left[ \frac{(1.104713)^{18} - 1}{0.104713} \right], A = $1,461.12 per year.

(d) \( k_4 = 10\% \) and monthly deposit: 
(i) \( N = 18 \times 12 = 216 \)  
(ii) Time Period = one month  
(iii) Periodic Rate: \( k_a = (1 + .1/4)^4 - 1 = 10.3813\% \)  
\[ \frac{k_{12}}{12} = (1.103813)_{12} - 1 = 0.8265\% \]  
FV of upfront deposit: $5,000(1 + .1/4)^{4 \times 18} = $29,586.14  
Required future amount = $100,000 – 29,586.14 = $70,413.86  
$70,413.86 = A \cdot \left[ \frac{(1.008265)^{216} - 1}{0.008265} \right], A = $118.35 per month.

5  
(a) \( k_a = 10\% \) and bi-annual deposit: 
(i) \( N = 50/2 = 25 \)  
(ii) Time Period = two years  
(iii) Periodic Rate: two-year rate = \( (1.1)^2 - 1 = 21\% \)  
\[ \text{FV} = 200 \cdot \left[ \frac{(1.21)^{25} - 1}{0.21} \right] = $110,848.43 \]  
\[ \text{PV} = 200 \cdot \left[ \frac{1 - (1.21)^{-25}}{0.21} \right] = $944.27 \]

(b) \( k_4 = 10\% \) and bi-annual deposit: 
(i) \( N = 50/2 = 25 \)  
(ii) Time Period = two years  
(iii) Periodic Rate: two-year rate = \( \left( 1 + \frac{0.1}{4} \right)^{4 \times 2} - 1 = 21.84\% \)  
\[ \text{FV} = 200 \cdot \left[ \frac{(1.2184)^{25} - 1}{0.2184} \right] = $126,882.41 \]
\[
PV = 200 \cdot \left[ \frac{1 - (1.2184)^{-25}}{0.2184} \right] = $909.19
\]

6 \hspace{1cm} N = 4 \times 10 = 40
(a) \hspace{1cm} k_a = 10\%
   (i) \hspace{1cm} N = 40
   (ii) \hspace{1cm} Time Period = one quarter
   (iii) \hspace{1cm} Periodic Rate: \quad \frac{k_a}{4} = (1.10)^\frac{1}{4} - 1 = 2.4114\%

\[
FV = 200 \cdot \left[ \frac{(1.024114)^{40} - 1}{0.024114} \right] = $13,218.59
\]
\[
PV = 200 \cdot \left[ \frac{1 - (1.024114)^{-40}}{0.024114} \right] = $5,096.33
\]
(b) \hspace{1cm} k_2 = 10\%
   (i) \hspace{1cm} N = 40
   (ii) \hspace{1cm} Time Period = one quarter
   (iii) \hspace{1cm} Periodic Rate: \quad k_a = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 10.25\%, \quad \frac{k_a}{4} = (1.025209)^\frac{1}{4} - 1 = 2.4695\%

\[
FV = 200 \cdot \left[ \frac{(1.024695)^{40} - 1}{0.024695} \right] = $13,389.69
\]
\[
PV = 200 \cdot \left[ \frac{1 - (1.024695)^{-40}}{0.024695} \right] = $5,046.44
\]
(c) \hspace{1cm} K_{12} = 10\%
   (i) \hspace{1cm} N = 40
   (ii) \hspace{1cm} Time Period = one quarter
   (iii) \hspace{1cm} Periodic Rate: \quad k_a = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 10.4713\%, \quad \frac{k_a}{4} = (1.010762)^\frac{1}{4} - 1 = 2.5209\%

\[
FV = 200 \cdot \left[ \frac{(1.025209)^{40} - 1}{0.025209} \right] = $13,542.25
\]
\[
PV = 200 \cdot \left[ \frac{1 - (1.025209)^{-40}}{0.025209} \right] = $5,003.19
\]

7 \hspace{1cm} A = $5,000 per year, N = 40.
(a) \hspace{1cm} Interest is compounded annually, k_a = 10\%:
   (i) \hspace{1cm} N = 40
   (ii) \hspace{1cm} Time Period = one year
   (iii) \hspace{1cm} Periodic Rate: one-year rate (k_a)
FV = 5,000 \cdot \left[ \frac{(1.10)^{40} - 1}{0.1} \right] = $2,212,963 \\
PV = 5,000 \cdot \left[ \frac{1 - (1.10)^{-40}}{0.1} \right] = $48,895.25 

(b) Interest is compounded quarterly, $k_4 = 10\%$: 
(i) $N = 40$ 
(ii) Time Period = one year 
(iii) Periodic Rate: one-year rate ($k_a$), $k_a = \left( 1 + \frac{0.1}{4} \right)^4 - 1 = 10.3813\%$

FV = 5,000 \cdot \left[ \frac{(1.103813)^{40} - 1}{0.103813} \right] = $2,455,276 \\
PV = 5,000 \cdot \left[ \frac{1 - (1.103813)^{-40}}{0.103813} \right] = $47,236.91 

(c) Interest is compounded continuously, $k_c = 10\%$: 
(i) $N = 40$ 
(ii) Time Period = one year 
(iii) Periodic Rate: one-year rate ($k_a$), $k_a = e^{0.1} - 1 = 10.5171\%$

FV = 5,000 \cdot \left[ \frac{(1.105171)^{40} - 1}{0.105171} \right] = $2,548,145 \\
PV = 5,000 \cdot \left[ \frac{1 - (1.105171)^{-40}}{0.105171} \right] = $46,670.90 

8 PVA = $1,000,000 with monthly withdrawal. 
(a) $k_2 = 8\%$ 
(i) $N = 25 \times 12 = 300$ 
(ii) Time Period = one month 
(iii) Period Rate: $k_a = \left( 1 + \frac{0.08}{2} \right)^2 - 1 = 8.16\%, \quad \frac{k_{12}}{12} = (1.0816)^{\frac{1}{12}} - 1 = 0.65582\%$

\[ $1,000,000 = A \cdot \left[ \frac{1 - (1.0065582)^{-300}}{0.0065582} \right], \quad A = $7,632.13 $ 

(b) $k_4 = 8\%$ 
(i) $N = 25 \times 12 = 300$ 
(ii) Time Period = one month 
(iii) Period Rate: $k_a = \left( 1 + \frac{0.08}{4} \right)^4 - 1 = 8.2432\%, \quad \frac{k_{12}}{12} = (1.082432)^{\frac{1}{12}} - 1 = 0.6623\%$

\[ $1,000,000 = A \cdot \left[ \frac{1 - (1.006623)^{-300}}{0.006623} \right], \quad A = $7,683.25 $
(c) $k_c = 8\%$

(i) $N = 25 \times 12 = 300$

(ii) Time Period = one month

(iii) Period Rate: $k_a = e^{0.08} - 1 = 8.3287\%, \quad \frac{k_{12}}{12} = (1.083287)^{\frac{1}{12}} - 1 = 0.66889\%$

$1,000,000 = A \cdot \left[ 1 - \left(1.0066889\right)^{-300} \right], \quad A = $7,735.88