Welfare Effects of Taxes in a Small Open Economy

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Abstract

Analyses of welfare effects of taxation policies typically start with a pristine setting without distortions and quantify the losses from tax increases. In reality, numerous taxes in existence distort the economy and changes in one tax instrument that would be welfare improving in their absence may lead to paradoxical losses. This paper provides quantitative examples of such paradoxes for a small open economy populated by infinitely-lived agents.

Keywords: small open economy, endogenous time preferences, adjustment costs, calibration, simulation, numerical methods, welfare paradoxes.

JEL Classification: C15, E17, B62

1 Introduction

Conventional tax incidence analysis tells us that increases in taxes will lead to a decrease in welfare. This analysis usually starts from a Pareto-optimal, competitive equilibrium, and frequently only one distortion is introduced at a time. According to Dixit[1] this is not a realistic approach. "The problem is that in reality we never find an economy that is pristine except for the one problem in which we are interested." Usually many taxes, subsidies, and other distortionary restrictions are in place simultaneously. Dixit uses the example of international trade theory concerned with "the need to find the best policy to counter the distortions or constraints..." For example, if we start from an

[1] Dixit, Avinash, Whither Greenwald-Stiglitz?, mimeo, Princeton University, 2003.

inefficient laissez-faire equilibrium because of some domestic distortion, a small trade tax or subsidy can yield a first-order welfare improvement, even if the instrument itself creates distortions of its own. This may result in "welfare paradoxes", namely taxes that improve long-run welfare rather than reduce it. The subject has been studied theoretically by Karayalcin (1995) in the context of of a small economy open to international asset trade . He studies the effects of various capital income tax policies. He shows that welfare paradoxes may exist if increases in tax instruments improve welfare. The purpose of this paper is to quantify the welfare effects of changes in tax rates in a small open economy. We conduct the simulation in the context of an intertemporal utility maximization framework. The paper applies numerical methods to the model developed by Karayalcin. We introduce changes in the tax rates and quantify both the impact on welfare, consumption and foreign assets, and the path to the new steady-state values. Section 2 lays out the model, Sections 3 summarizes the theoretical results, Section 4 presents the calibration of the model, and Section 5 concludes.

2 The Model

We start with a small open economy populated by a constant number of identical and infinitely-lived households with perfect foresight. For the sake of simplicity we normalize the size of the households to one. This economy is also home of perfectly competitive firms that produce a single good that can be used either for consumption or investment. A detailed analysis of households and firms follows.

2.1 Households

Each household supplies exactly one unit of labor at a wage rate w_t per unit of time. They also receive interest income from their non-human wealth a_t. Interest income is defined as $r(1-\tau_r)a_t$, where r is the wold real rate of interest and τ_r is personal tax on interest income. We assume that the world interest rate is constant. Households have variable rates of time preference as in Epstein and Hayes (1983) and they maximize lifetime welfare U over the consumption path C. The maximization problem is:

$$U(C) = -\int_0^\infty \exp(-z_t)\exp\left[-(1-\tau_r)rt\right]dt \tag{1}$$

subject to

$$\dot{z}_t = u(c_t) - (1-\tau_r) r, \tag{2}$$

$$\dot{a} = (1-\tau_r) r_t + w_t - c_t + \tau_t, \tag{3}$$

$$z_0 = 0, \tag{4}$$

where $u(c) > 0$ is the felicity function with $u' > 0$, $u'' < 0$, and τ_t is a lump-sum government transfer. We assume the government runs a balanced budget so that the lump-sum transfer is equal to the net tax revenue.

The lifetime welfare function U differs from the conventional time-additive utility functions because it is recursive. This implies that the marginal rate of substitution between times t and s $(s > t)$ is independent of consumption before t but no after s. This gives rise to a variable rate of time preference Ω at time s:

$$\Omega_s = \left\{ \int_0^{\infty} \exp\left[- \int_s^t u(c)\, d\tau \right] dt \right\}^{-1}. \tag{5}$$

Ω at s is the following function of the utility function $U(C)$

$$\Omega(\phi_s) = -\phi^{-1}, \quad \phi_s = U({}_sC), \tag{6}$$

where ${}_sC$ stands for that part of the consumption path C beyond time s, and ϕ_s denotes lifetime welfare at time s. When the consumption path is globally constant, as in long-run equilibrium, $U({}_sC) = -1/u(c)$, and the rate of time preference is given by

$$\Omega^*(\phi^*) = u(c^*), \tag{7}$$

where the stars indicate long-run equilibrium.

The assumption $u' > 0$ implies that Ω is increasing in the consumption path C. This assumption is known as 'increasing marginal impatient' (see Lucas and Stokey, 1984). There are several arguments in support of it. For us is sufficient to say that local stability would fail to obtain in the absence of that assumption.

The felicity function can be expressed as

$$u(c_t) = \omega + \ln c_t, \tag{8}$$

where ω is a parameter measuring generalized time preference.

It can be shown that with equation (8) the solution to the lifetime welfare maximization problem yields

$$\dot{c} = [(1-\tau_r)\,r - \Omega(\phi)]\,c. \tag{9}$$

The equation of motion of lifetime welfare ϕ is obtained by differentiating (1) with respect to time:

$$\dot{\phi} = 1 + u(c)\,\phi. \tag{10}$$

2.2 Firms

Firms in the model are perfect competitors that employ capital, k, and labor, l, to produce a single good using a constant returns to scale production technology. New investment is financed by issue of corporate bonds, b^c, and by retained earnings. Profits not used to finance investment, net of corporate income tax, are paid to shareholders as dividends. Firms deduct interest payments on outstanding debt as well as adjustment costs T associated with investment when determining taxable corporate profits. Total dividends before personal tax, π, are given by

$$\pi \equiv [f(k) - w - rb^c - T]\,(1-\tau_c) + \dot{b^c} - (1-\tau_l)\,i, \tag{11}$$

where τ_c is the corporate income tax and τ_l is the rate of investment credit. $f(k)$ is a constant returns to scale production function that satisfies the standard Inada conditions.

In the absence of uncertainty we cannot adequately account for the differences in the forms of financing. Therefore we assume that the representative firm finances a fraction ε of new investment from retained earnings, and a fraction $(1-\varepsilon)$ of it issuing corporate bonds. Thus,

$$b^c = (1-\varepsilon)\,k, \quad \dot{b^c} = (1-\varepsilon)\,\dot{k} \tag{12}$$

Installing investment goods is costly. It takes $i\,[1 + T(i/k)]$ units of output to increase the stock of capital by i units. The installation-cost function T has the following properties:

$$T(0)=0, \quad T'(\cdot)>0, \quad 2T'+(i/k)\,T''>0. \tag{13}$$

Corporate bonds, b^c, foreign bonds, b^f, and equities are perfect substitutes in the portfolio of households. If E denotes the market value of outstanding equity then the arbitrage condition can be stated as:

$$r(1-\tau_r)=\frac{\pi}{E}+\frac{(1-\tau_g)\,\dot{E}}{E} \tag{14}$$

where τ_g stands for the tax rate on accrued capital gains. This arbitrage condition must hold at all times. The term on the left hand side of (14) represents the after-tax return on foreign bonds, while the expression on the right hand side denotes the after-tax return on equity. This expression is the after-tax sum of current yield and capital gains. Using (11) and (12) in (14) and integrating, we obtains the market value of equity at time 0:

$$E=\int_0^{\infty}\theta_g^{-1}\pi\exp\left(-r\theta_r\theta_g^{-1}t\right)dt, \quad \theta_j\equiv 1-\tau_j, \quad j=c,g,r. \tag{15}$$

The representative firm chooses the time path of investment by maximizing the market value of E subject to the constraint $i=k$. This yields

$$\dot{q}=r\theta_r\theta_g^{-1}-\theta_c\theta_g^{-1}\left[f'(k)-r(1-\varepsilon)-\left(\frac{i}{k}\right)^2T'\left(\frac{i}{k}\right)\right], \tag{16}$$

$$q=\theta_g^{-1}\left\{(\varepsilon-\tau_l)+\theta_c\left[T+\left(\frac{i}{k}\right)T'\left(\frac{i}{k}\right)\right]\right\}, \tag{17}$$

$$w=f(k)-f'(k)\,k, \tag{18}$$

where q denotes the shadow value of capital and can be easily shown to be equal to the stock market price of one unit of equity relative to the replacement cost of capital (that is to say, q of Tobin's Q). Substituting (17) into (16), this last equation is an arbitrage equation setting the after-tax rate of return on foreign bonds to the after-tax rate of return on equity. Equation (18) is the familiar condition that requires that the marginal productivity of labor be equal to the real wage rate. Equation (17) implies that the rate of investment, i/k, is the following function of q

$$\frac{i}{k}=\frac{\dot{k}}{k}=\varphi(q), \quad \varphi'(q^*)>0, \tag{19}$$

which states that the rate of investment is an increasing function of 'Tobin's Q'. The most prominent feature of (16) and (19) is that neither q nor investment depends on the consumption and savings decisions of households. Equations (16) and (19) constitute a system of two autonomous differential equations in q and k.

2.3 The current account

To obtain the dynamics of the current account we use equations (3), (11)-(19), $a = b^f + qk + b^c$, and recall the assumption of balanced government budget to obtain

$$\dot{b^f} = rb^f + f(k) - i(1 - T) - c, \tag{20}$$

The current account is equal to output plus interest earnings on foreign assets minus consumption and investment spending.

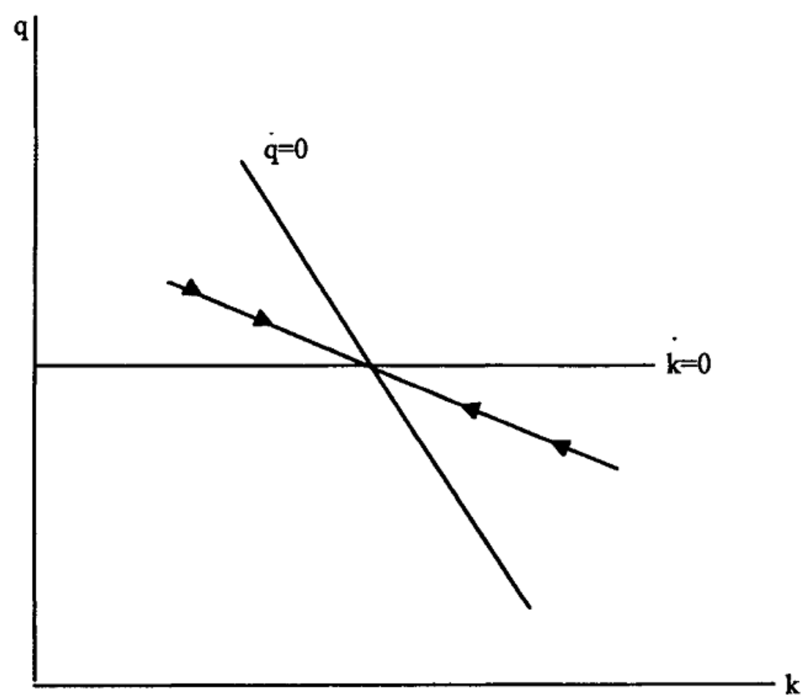


Figure 1. The adjustment of q and k.

2.4 Equilibrium

The model has five differential equations: (9), (10), (16), (19) and (20). To obtain the steady-state value of $x = x(q, k, c, \phi, b)$ set these equations to zero. This yields

$$q^* = \theta_g^{-1}\left(\varepsilon - \tau_l\right), \quad q^* = \overline{q}\left(\tau_g, \tau_l\right), \overline{q}_1 > 0, \overline{q}_2 < 0, \tag{21}$$

$$\begin{aligned} f'\left(k^*\right) &= r\left[\left(1-\varepsilon\right) + \theta_r\theta_c^{-1}\theta_g^{-1}\left(\varepsilon - \tau_l\right)\right] = \widetilde{r}, \quad k^* = \overline{k}\left(\tau_g, \tau_c, \tau_r, \tau_l\right), \\ \overline{k}_l &< 0 \quad \left(i = 1, 2\right), \quad \overline{k}_j > 0 \quad \left(j = 3, 4\right), \end{aligned} \tag{22}$$

$$rb^{f*} + f\left(k^*\right) = c^*, \quad \overline{b}\left(\tau_g, \tau_c, \tau_r, \tau_l\right), \quad \overline{b}_i > 0 \ \left(i = 1, 2\right), \quad \overline{b}_j < 0 \ \left(j = 3, 4\right) \tag{23}$$

$$u\left(c^*\right) = r\theta_r, \quad c^* = \overline{c}\left(\tau_r\right), \quad \overline{c}' < 0, \tag{24}$$

$$\phi^* = -\left(r\theta_r\right)^{-1}, \quad \phi^* = \overline{\phi}\left(\tau_r\right), \quad \overline{\phi}' < 0, \tag{25}$$

We assume in (21) that $\varepsilon > \tau_l$ so that the equity price q is positive in the steady state. Equation (22) shows that capital income taxes and the investment tax credit drive a distortionary wedge between the marginal productivity of capital $f'(k)$ and the world real interest rate r. Whether this wedge causes the marginal productivity of capital to exceed the world interest rate or not depends on the values of the tax rates τ_g, τ_c, τ_r, and the investment tax credit τ_l:

$$f'\left(k*\right) \lesseqgtr r \Longleftrightarrow \left\{1 - \left[\frac{\tau_l}{\varepsilon} + \frac{\theta_c\theta_g}{\theta_r}\right]\right\} \lesseqgtr 0. \tag{26}$$

If income taxes are uniform ($\tau_g = \tau_r$), the corporate tax rate is fully integrated ($\tau_c = 0$), and there is no investment credit ($\tau_l = 0$), it then follows form (22) that $f'(k^*) = r$. In other words, income tax is neutral with regards to investment. Additionally, (17) indicates that in the absence of the investment credit and the tax on capital gains $q = \varepsilon$. Therefore, in this case, the market value of equity $E - qk = \varepsilon k$ equals the equity-financed portion of the accumulated investment of firms. However, actual tax systems do not generally have fully integrated corporation taxes, nor are the rates of investment tax credit set equal to zero. It is therefore useful to look at some representative tax rates for such countries as Canada, Sweden, and the Netherlands, which are generally considered to be small open economies, to have an understanding of the difference between $f'(k^*)$ and r as implied by the rates in (26). The effective average rates are roughly: $\tau_r = 0.45$,

$\tau_c = 0.25$, $\tau_l = 0.10$, $\tau_g = 0.27$; in addition $\varepsilon = 0.75$. These values imply that $f'(k^*) < r$ for the economies in question.

Notice from (23)-(25) that the variable rate of time preference implies well-defined long-run levels of target utility, consumption and wealth.

At any point in time, given the parameters and steady-state level of $x = (q, k, c, \phi, b)$ the five differential equations solve for x. Equation (18) determines the wage rate.

The dynamic system ((9), (10), (16), (19), (20)) possesses two negative (λ_1 and λ_2) and three positive eigenvalues, which (given the two predetermined variables k and b^f) render a stable saddle path. Its motion along the convergent saddle-path is characterized by

$$k_t - k^* = (k_0 - k^*) \exp(\lambda_1 t), \tag{27}$$

$$q_t - q^* = [\lambda_1 / k^* \varphi'(\overline{q})](k_t - k^*), \tag{28}$$

$$c_t - c^* = (r - \lambda_2)\left[\left(b_0^f - b^{f*}\right) + \mu(k_0 - k^*)\right] \exp(\lambda_2 t), \tag{29}$$

$$\phi_t - \phi^* = \beta(c_t - c^*), \tag{30}$$

$$b_t^f - b^{f*} = -\mu(k_0 - k^*) \exp(\lambda_1 t) + \left[\left(b_0^f - b^{f*}\right) + \mu(k_0 - k^*)\right] \exp(\lambda_2 t), \tag{31}$$

where

$$\mu \equiv [f'(k^*) - \lambda_1][r - \lambda_1]^{-1} > 0, \qquad \beta \equiv [rc^*\theta_r(r\theta_r - \lambda_2)]^{-1} > 0.$$

In the perfect foresight, intertemporal-equilibrium framework adopted in this paper the dynamics of the variables are determined by the long-run changes they go through. By this token, (27) indicates that along the convergent path the dynamics of investment are uniquely determined by its speed of adjustment and the long-run change in the domestic capital stock, while (31) shows that the dynamics of the current account depends on long-run changes in both foreign assets and the domestic capital stock, as well as the adjustment speed of both.

Straightforward manipulation of (27) and (31) yields

$$\dot{k} = \lambda_1(k_t - k^*), \tag{32}$$

$$\dot{b^f} = \mu(\lambda_2 - \lambda_1)(k_t - k^*) - \lambda_2\left(b_t^f - b^{f*}\right). \tag{33}$$

It follows from (32) and (33) that if $\lambda_1 < \lambda_2$ the capital stock, k, will adjust faster than the foreign assets holdings, b^f. In other words, the current account will be more persistent than domestic investment. This, however, contradicts the findings of recent studies. Consequently, we will concentrate on the case $\lambda_1 > \lambda_2$.

Further, since $\beta > 0$, (29) and (30) indicate that consumption c and lifetime welfare ϕ always rise and fall together along the convergent path shown in Figure 2.

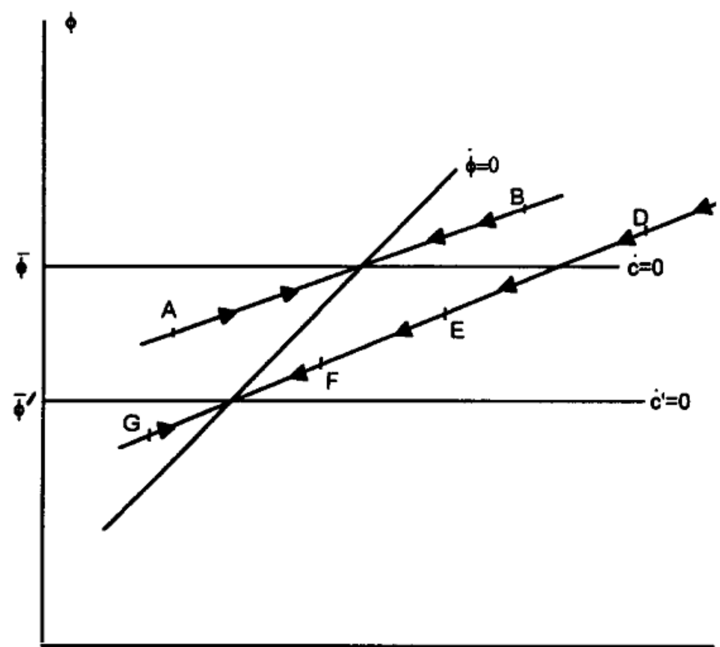


Figure 2. The adjustment of consumption and lifetime welfare.

3 The effects of taxation

We now need to put the model to work to assess the effects of changes in the tax rates. First we will briefly present the expected results from the theoretical model. Later, in Section 4, we will calibrate the model to evaluate the effects of changes in tax instruments on welfare. We start analyzing the effects of changes in the investment tax credit τ_l, in the tax rates on capital gains τ_g, and on corporate income τ_c. They are treated jointly because changes in these rates do not affect the long-run target utility of households (equation (24)), and therefore do not have any effects on the long-run level of consumption.

Let us start by analyzing the effects of an unanticipated permanent increase in the corporation tax rate (τ_c). On impact this reduces dividends and the rate of return on equity. The consequent incipient excess stock demand for foreign assets leads to an immediate drop in the price of equity q (and to expectations of capital gains), which starts a process of capital decumulation. The process continues until the decline in the capital stock pushes the marginal productivity of capital and the dividends up sufficiently enough so that, in the absence of expectations of capital gains on it, the rate of return on equity equals the rate of return on foreign bonds.

Likewise, an unanticipated permanent increase in the tax rate on capital gains (τ_g) reduces the long-run stock of capital. This raises the steady-state marginal productivity of capital and dividends, therefore the long run equity price q must also rise to ensure the equality of assets yields. The short-run adjustment of the forward looking equity-price q is driven by the long-run changes in k and q. This puts opposing pressures on q and as a consequence the impact effect of the increase in τ_g on q is ambiguous. However, regardless of whether it drops or jumps on impact, along the convergent path, q will rise towards its higher steady-state level (equation (28)).

On the contrary, an unanticipated permanent increase in the investment tax credit τ_l also increases the long-run stock of capital by reducing the effect of the replacement cost of capital. The resulting decrease in the marginal productivity of capital and dividends requires a long-run fall in q to raise the rate of return on equity and to ensure the equality of assets yields. As in the case of the rise in τ_g, the opposing pressures these long-run changes have on q may result in a drop or jump of the equity price on impact. However, q unambiguously falls in the medium-run along the adjustment path.

To see the consequences of these policies on consumption and lifetime welfare we use (22) and (23) in (29) and (30) and set $t = 0$ to obtain

$$\phi_0 - \phi^* = \beta \left(c_0 - c^* \right), \tag{34}$$

$$c_0 - c^* = \left(-\lambda_1 \right) \left(r - \lambda_2 \right) \left(r - \lambda_1 \right)^{-1} \overline{k}_j \left[f' \left(k^* \right) - r \right], \qquad j = 1, 2, 4, \tag{35}$$

which show the changes in consumption and lifetime welfare on impact. It is important to remember that the long-run levels of c and ϕ remain the same. Note that ϕ_0 is the present discounted value of the future felicity stream at time $t = 0$ and ϕ^* is unaffected by these policies. Therefore the sign of $\phi_0 - \phi^*$ directly measures the welfare effects of the policies under consideration. Additionally, since $\beta > 0$, the sign of $\phi_0 - \phi^*$ is the same as the sign of $c_0 - c^*$.

To understand the changes in consumption c on impact recall that households try to attain the target utility level $u(c^*)$, which is not affected by the two policies under consideration. The implication is that the long-run level of consumption and the level of wealth and income required to support it also remain constant. Faced with these policies, foreward-looking households choose transition paths that allow them to attain the original target utility level.

We will now consider the effects of the increase of the tax rate on corporate income (τ_c). This decreases the economy's capital stock (22) and GDP. To attain the original level of utility households must increase their long-run holdings of foreign assets (23). On impact, the long-run decrease in the domestic capital stock makes a short-run consumption binge possible. But the required long-run increase in the foreign assets holdings demands an immediate increase in savings and a drop in consumption on impact. If the long-run effect of the decrease in the stock of capital is such that $f'(k^*) > r$ holds, then consumption must drop on impact which implies that $c_0 - c^* < 0$ in equation(35). Consumption is at point A in Figure 2. In the opposite case c will jump on impact to point B in Figure 2. Welfare (ϕ_0) follows consumption (c_0), so if consumption jumps on impact lifetime welfare will jump too. There is an alternative and useful way of looking at this result. As we saw, the direction of the change in lifetime welfare do to a change in τ_c depends on the sign of $f'(k^*) - r$. We know that the difference between the marginal productivity of capital and the world real interest rate is caused by the presence of distortionary taxes. If $f'(k^*) > r$ initially, the fall in the stock of capital caused by the increase in τ_c will accentuate the distortion and reduce lifetime welfare by increasing the marginal productivity of domestic capital. On the other hand, if $f'(k^*) < r$ initially, the same policy will reduce the distortion and increase lifetime welfare.

The increase in τ_g has similar long-run effect on the domestic stock of capital and the foreign assets holdings (22), (23) as the increase in τ_c, therefore it will have similar consequences for consumption and lifetime welfare for the same reason.

The increase in the investment tax credit (τ_l) has the opposite consequences for k^* and b^{f*}: it increases the long-run domestic stock of capital and reduces the long-run foreign asset holdings (22), (23). Thus, if initially $f'(k^*) > r$, the increase in τ_l will decrease the marginal productivity of domestic capital, reduce the distortion, and rise lifetime welfare. However, given the tax rates we have considered we will have $f'(k^*) < r$, and a reduction in lifetime welfare will seem more plausible.

We now analyze the effects of an increase in the tax rate on personal interest income (τ_r). Unlike the former policies this one changes the long-run target level of utility. An anticipated permanent increase in τ_r reduces the rate of return on foreign bonds. On impact, this creates an incipient excess stock demand for equity, which is eliminated by an immediate jump in the price q of equity that lowers the yield on it. The subsequent rise in investment in the medium run will increase the domestic stock of capital, and decrease its marginal productivity until the equity price q and the rate of investment return to their initial levels in the long-run.

It is also worth noting that, given the increase in the domestic stock of capital and the decrease in their long-run target utility level, the rise in τ_r will lead forward-looking households to decrease their foreign asset holdings (23) across steady-states.

Finally we briefly discuss the motion of the current account along the convergent path. Since the underlying logic is basically the same as in all the exercises presented above, we will focus on the effects of an increase in the tax rate on capital gains (τ_g). Although the foreign asset holdings of households rise in the long run, initially the economy runs a current account deficit. To see why consider the following. On impact, the drop in the equity price q leads to an immediate decrease in investment, which by itself would give rise to a current account surplus. However, we also saw that consumption c may jump or drop on impact. If c jumps sufficiently enough to outweigh the drop in investment, domestic absorption may in fact initially rise, causing a current account deficit. As the economy increases its long-run holdings of foreign assets, it must, however, run a current account surplus later. This implies a non-monotonic adjustment of the current account. On the other hand, if c drops on impact, or if the jump in c is outweighed by initial domestic capital decumulation, the economy will run a current account surplus on impact, and will adjust monotonically.

4 Calibration of the Model

With the theoretical model in place, we proceed to numerically calibrate it. Table 1 summarizes the information about the starting values of the main variables.

τ_r	τ_c	τ_g	τ_l	r
0.45	0.25	0.27	0.10	0.06
$\theta_r = (1 - \tau_r)$	$\theta_c = (1 - \tau_c)$	$\theta_g = (1 - \tau_g)$	ε	ω
0.55	0.75	0.73	0.75	-0.59

Table 1

To estimate GDP we use a CES production function of the form

$$f(k,l) = A\left[\delta k^{-\rho} + (1-\delta)\, l^{-\rho}\right]^{-\frac{1}{\rho}}$$

where the values of the parameters are: $A = 1.10746$; $\delta = 0.4$ and $\rho = 0.5$. The size of the labor force l is normalized to one. The value of A comes from equation (22), $f'(k^*) = \widetilde{r}$, and δ, ρ, and r are calibrated to fit stylized facts and to deliver a meaningful steady state.

With the starting values and the equilibrium equations stated in (21)-(25) we obtain the following steady-state value of $x^* = x^*\left(q^*, k^*, c^*, \phi^*, b^{f*}\right)$

q^*	k^*	c^*	ϕ^*	b^{f*}
0.89411	7.24145	1.86451	-30.303	-1.85744
Table 2				

In order to calculate the saddle-path equilibrium we start with the system of equations (9), (10), (16), (19), and (20). Once linearized, the system yields the following result:

$$\dot{x} = M\left(x - \bar{x}\right) \tag{36}$$

where

$$M = \begin{bmatrix} r\theta_r\theta_g^{-1} & -\theta_c\theta_g^{-1}f''(k^*) & 0 & 0 & 0 \\ k\varphi'(q) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c\left(r\theta_r\right)^2 & 0 \\ 0 & 0 & -\left(rc\theta_r\right)^{-1} & r\theta_r & 0 \\ -k\varphi'(q) & f'(k) & -1 & 0 & r \end{bmatrix}$$

and $x = x\left(q, k, c, \phi, b^f\right)$. From M we can calculate the eigenvalues of the system. There are three positive and two negative values. The negative eigenvalues are:

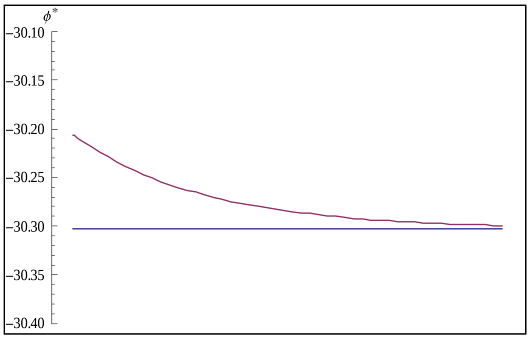
$$\begin{aligned} \lambda_1 &= \left(\frac{1}{2}\right)\left\{r\theta_r\theta_g^{-1} - \sqrt{\left(r\theta_r\theta_g^{-1}\right)^2 - 4f''(k)\,k\varphi'(q)\,\theta_c\theta_g^{-1}}\right\} \\ \lambda_2 &= \left(\frac{1}{2}\right)\left\{r\theta_r - \sqrt{\left(r\theta_r\right)^2 + 4r\theta_r}\right\}. \end{aligned} \tag{37}$$

The values of the negative eigenvalues corresponding to x^* are respectively $\lambda_1 = -1.78214$ and $\lambda_2 = -0.165907$

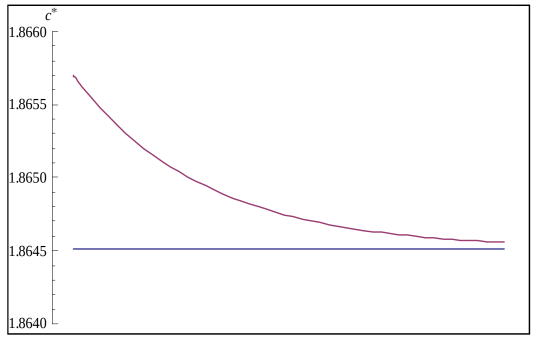
4.1 The impact of changes in tax rates on welfare

Now we investigate the effects that changes in the different taxes have on welfare. We consider both the change on impact and the new steady-state value.

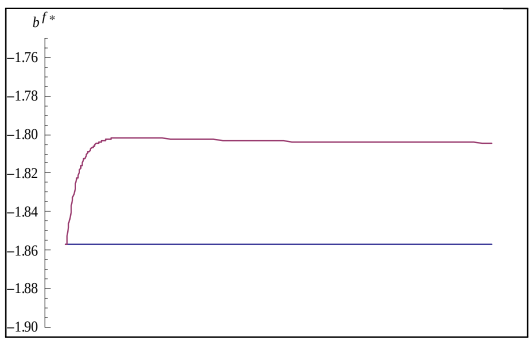
4.1.1 Increase in corporate tax rate



Welfare



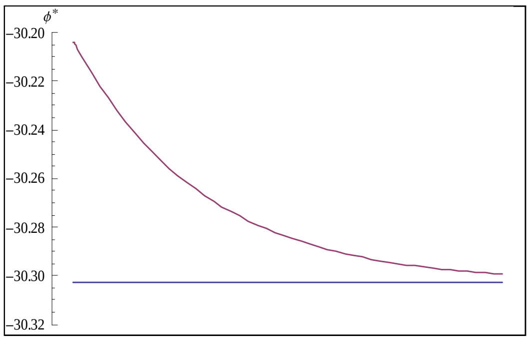
Consumption



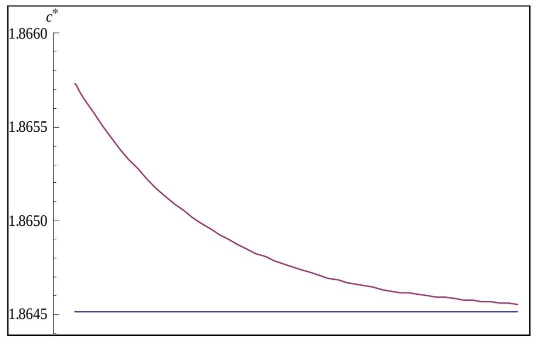
Foreign Assets

We now consider the effects of an increase in the tax rate on corporate income (τ_c). Whether the rise in τ_c increases lifetime welfare or not depends on the sign of $f'(k^*) - r$. We must recall that the difference between the marginal productivity of capital and the world real interest rate is caused by the presence of distortionary taxes. If initially $f'(k^*) > r$, the fall in the stock of capital caused by the rise in τ_c will accentuate the distortion and reduce lifetime welfare. On the other hand, if $f'(k^*) < r$ initially the same policy will reduce the distortion and increase lifetime welfare. The results of our experiment do not exactly match the predictions of the theory. Consumption and welfare do jump on impact in response to an increase of one percentage point in the tax (0.064% and 0.32% respectivly), but since the long-run level of consumption required to support the target utility level $u(c^*)$ is not affected by this tax policy, eventually both consumption and welfare drift towards theirs initial values. Foreign assests on the other hand improve by 2.85%.

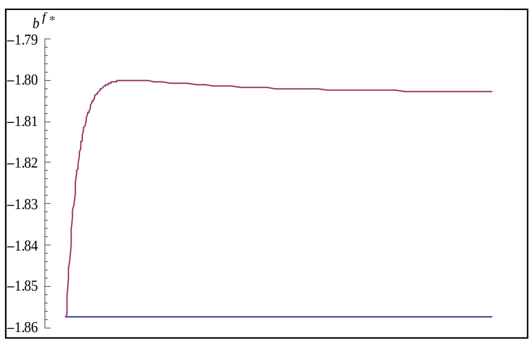
4.1.2 Increase in the rate on accrued capital gains



Welfare



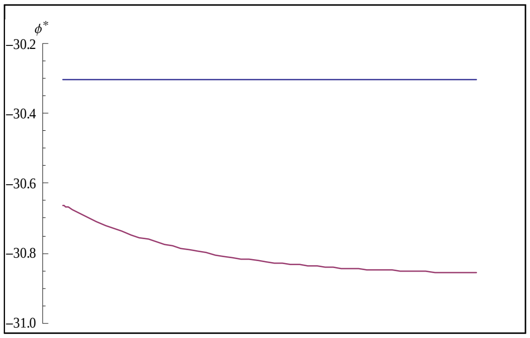
Consumption



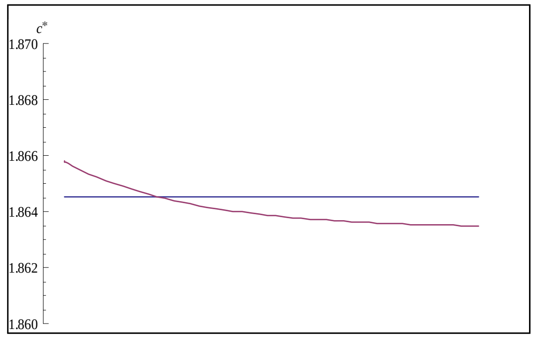
Foreign Assets

An increase in τ_g has similar long-run effects on the domestic stock of capital and the foreign assets holdings ((22) and (23)) as an increase in τ_c. Therefore it will have similar effects on consumption and lifetime welfare for the same reasons. Welfare jumps on impact 0.33% do to an increase of one percentage point in the tax policy rate. Consumption also jumps on impact, but only 0.065%. However both return to the initial steady-state level as the tax under consideration does not affect the long-run target utility level and welfare. Foreign assets find a new steady-state 2.93% above the original level.

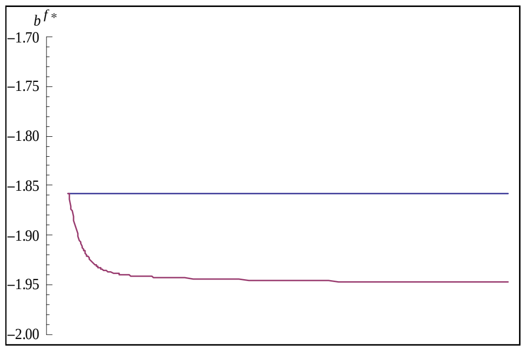
4.1.3 Increase in personal income tax rate



Welfare



Consumption



Foreign Assets

The theoretical model tells us that an increase in the tax rate on personal interest income (τ_r) changes the long-run target level of utility. With the increase in τ_r households' long-run utility target falls and consequently they have to reduce their long-run consumption. To see the effect of this policy on households' lifetime consumption and welfare we can rewrite equations (34) and (35) incorporating the long-run changes in c and ϕ:

$$c_0^+ - c_0^- = (-\lambda_1)(r-\lambda_2)(r-\lambda_1)^{-1}\bar{k}_3\left[f'(k^*)-r\right] - \lambda_2 c^*, \tag{38}$$

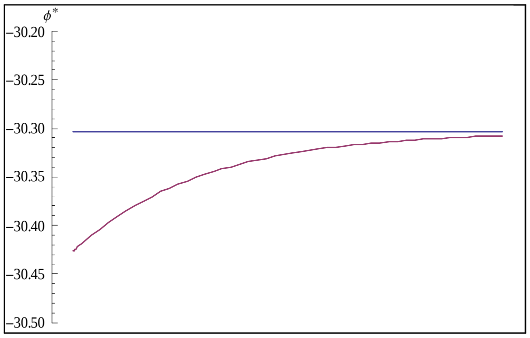
$$\phi_0^+ - \phi_0^- = \beta\left\{(-\lambda_1)(r-\lambda_2)(r-\lambda_1)^{-1}\bar{k}_3\left[f'(k^*)-r\right] + (r-\lambda_2)c^*\right\} - (\theta_r^n\theta_r)^{-1}(\tau_r^n - \tau_r), \tag{39}$$

where superscripts (-) and (+) refer to the values of the variables before and after the discrete change on impact and superscript n denotes the new levels of τ_r and θ_r. Equation (38) tells us that if initially the marginal productivity of capital exceeds the world real interest rate (r), consumption will jump on impact. However we cannot infer from the jump that lifetime welfare increases. The reason is that although an initial jump in c increases welfare, consumption decreases across steady-states and as a consequence the new long-run level of welfare must be lower. Still, as equation (38) indicates, $f'(k^*) > r$ is a necessary but not sufficient condition for welfare to improve. For small changes in the tax rate, the reduction in the distortion implied by an initial $f'(k^*) > r$ and the rise in the capital stock will raise lifetime welfare. In the empirically more plausible case, consumption and lifetime welfare may drop on impact.

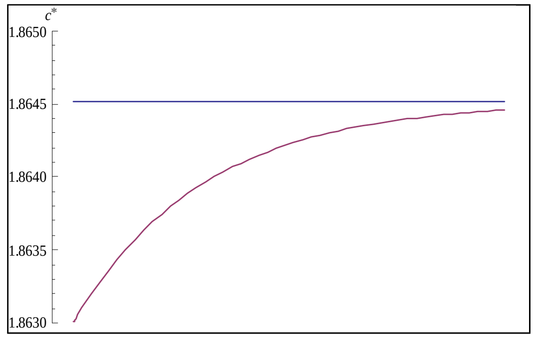
The results of the simulation broadly coincide with what the theory predicts. Nevertheless there are some paradoxes. Although we have $f'(k^*) < r$ before and after[2] the increase in τ_r, consumption jumps 0.07% on impact in response to an increase of one percentage point in the tax rate. Eventually the new long-run level of consumption drifts towards a value below the initial steady-state by 0.06%. Welfare on the other hand drops on impact 1.19% and continues to decrease until it reaches its new steady-state level, 1.85% below the initial one. Foreign assests do not jump or drop on impact. They drift to a new steady-state value, 4.87% below the initial one.

4.1.4 Increase in the rate of investment credit

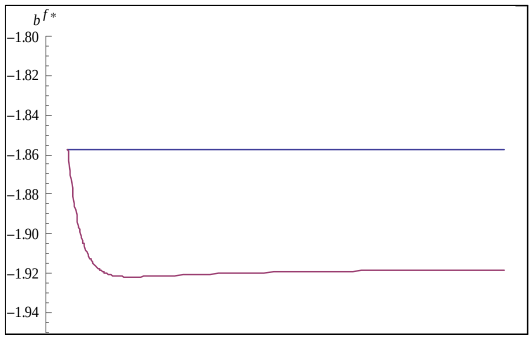
[2]The values of $f'(k^*)$ before and after are respectively 0.054 and 0.053.



Welfare



Consumption



Foreign Assets

We now consider the effects of an increase in the investment tax credit (τ_l). We can apply the same logic as in the case of an increase in τ_c, only this time the effects are reversed. Whether the rise in τ_l increases lifetime welfare or not also depends on the sign of $f'(k^*) - r$. As we mentioned before, the difference between the marginal productivity of capital and the world real interest rate is caused by the presence of distortionary taxes. If initially $f'(k^*) > r$, the increase in the stock of capital caused by the rise in τ_l will reduce the distortion and increase lifetime welfare. On the other hand, if $f'(k^*) < r$ initially, the same policy will increase the distortion and decrease lifetime welfare. The numerical values of our exercise tell us that our case is the latter. Consumption and welfare drop on impact by 0.08% and 0.41% respectively. They increase afterwards and eventually reach theirs original steady-state level. Foreign assets find a new steady-state level 3.27% below the original one.

5 Conclusions

We presented a model of a small open economy where households have endogenous time preferences and investment is subject to adjustment costs. We used the theoretical model to predict the effects of changes in various tax rates considered. Later we calibrated the model selecting a set of plausible values for the variables and running numerical simulation to estimate the effects of different tax policies. The quantitative results generally coincide with the prediction of the theory. These results are affected by differences between the marginal productivity of capital and the world real interest rate. Depending on the sign of the difference between the two, increase in taxes may lead to decreases or increase in lifetime welfare, giving way to welfare paradoxes. This results stem from the fact that the economy is distorted by the presence of taxes. Increases in taxes may sometimes counteract some of those distortion resulting in a welfare increase instead of the expected decrease. Hence the welfare paradox.

6 Appendix

Steady-state values of the variables in the model after an increase of 1% in each tax rate.

Variables	ϕ^*	k^*	q^*	c^*	b^{f*}	λ_1	λ_2
Initial Values	-30.3030	7.24145	0.890411	1.86451	-1.85739	-1.78214	-0.165907
1% increase in τ_r	-30.8642	7.32157	0.890411	1.86339	-1.94495	-1.77162	-0.164528
1% increase in τ_c	-30.3030	7.18309	0.890411	1.86451	-1.80449	-1.79020	-0.165907
1% increase in τ_l	-30.3030	7.30912	0.876712	1.86451	-1.91816	-1.77290	-0.165907
1% increase in τ_g	-30.3030	7.18149	0.902778	1.86451	-1.80297	-1.79012	-1.65907

Effects of tax policies on welfare, consumption and foreign assets

a) One percentage point increse in personal tax rate income (τ_r)

Variables	Initial Value	Jump in impact	%Δ	New Steady-State	%Δ
Welfare	-30.3030	-30.6626	1.19	-30.8642	1.85
Consumption	1.86451	1.86579	0.069	1.86339	-0.06
Foreign assets	-1.85739	-1.85739	0	-1.9479	4.87

b) One percentage point change in corporate tax rate (τ_c)

Variables	Initial Value	Jump on impact	%Δ	New Steady-State	%Δ
Welfare	-30.3030	-30.2063	-0.32	-30.3030	0
Consumption	1.86451	1.8657	0.064	1.86451	0
Foreign assets	-1.85739	-1.85739	0	-1.80443	-2.85

c) One percentage point change in investment tax credit (τ_I)

Variables	Initial Value	Jump on impact	%Δ	New Steady-State	%Δ
Welfare	-30.3030	-30.4262	0.41	-30.3030	0
Consumption	1.86451	1.86301	-0.08	1.86451	0
Foreign assets	-1.85739	-1.85739	0	-1.91816	3.27

d) One percentage point change in tax on accrued capital gains (τ_g)

Variables	Initial Value	Jump on impact	%Δ	New Steady-State	%Δ
Welfare	-30.3030	-30.2038	-0.33	-30.3030	0
Consumption	1.86451	1.86573	0.065	1.86451	0
Foreign assets	-1.85739	-1.85739	0	-1.80297	-2.93

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