1. (a) Let $X$ be a set with more than one element. Show that there exists a metric $d$ on $X$, so that the topology generated by $d$ is the discrete topology on $X$.

(b) Let $X$ be a set with more than one element. Show that for no metric $d$ on $X$, the topology generated by $d$ is the same as the trivial topology on $X$.

(c) Let $X = \{a, b, c\}$. Give an example of a topology on $X$, other than the trivial topology, that cannot be the topology generated by a metric on $X$.

2. (a) Consider $\mathbb{R}_l$, that is, $\mathbb{R}$ endowed with the lower limit topology (see notes or textbook). Show that a sequence $\{x_n\}_n$ converges to $x$ in the lower limit topology if and only if $\{x_n\}_n$ converges to $x$ in the standard topology and $x_n \geq x$ for all except finitely many values of $n$.

(b) Consider $\mathbb{R}_K$, that is, $\mathbb{R}$ endowed with the topology $T_K$ (see notes or textbook). Formulate and prove the equivalent condition for the convergence of a sequence in $\mathbb{R}_K$. 