5. page 92. (a) If \( f \) is differentiable at \( a \) then \( f \) is continuous at \( a \). Since \( f(a) \neq 0 \), it follows from a result done in Chapter 3 (see lemma 3.28, or Ex. 4, p. 78) that \( f(x) \neq 0 \), for any \( x \) in an interval \( I = (a - \delta, a + \delta) \) around \( a \). Thus for any \( h \in (-\delta, \delta) \), \( f(a + h) \neq 0 \). □

(b) We need to show that the limit
\[
\lim_{h \to 0} \frac{\frac{1}{f(a+h)} - \frac{1}{f(a)}}{h}
\]
exists, and is equal to \(- \frac{f'(a)}{f^2(a)}\).

As \( f(a) \neq 0 \), we have that \( \frac{1}{f(a)} \) is well defined and, from part (a), also \( \frac{1}{f(a+h)} \) is well defined for \( h \) small enough. Then
\[
\lim_{h \to 0} \frac{\frac{1}{f(a+h)} - \frac{1}{f(a)}}{h} = \lim_{h \to 0} \left( \frac{f(a) - f(a + h)}{h} \cdot \frac{1}{f(a)f(a+h)} \right) = - \frac{f'(a)}{f^2(a)},
\]
where the first equality is obtained after elementary algebra and the second follows from the definition of the derivative at \( a \) and the fact that \( f \) is also continuous at \( a \). Thus
\[
\left( \frac{1}{f} \right)'(a) = - \frac{f'(a)}{f^2(a)}. \quad □
\]

5. page 100. (a) Let \( x \in \mathbb{R} \setminus 0 \) arbitrary. The conditions to apply the Mean Value Theorem for \( f \) on the interval between 0 and \( x \) are satisfied, so there exists \( y \) (between 0 and \( x \)) such that
\[
f(x) - f(0) = f'(y)(x - 0).
\]
Thus, from the assumption, it follows that \( f(x) - f(0) = 0 \), for all \( x \in \mathbb{R} \). □

(b) The inequality trivially holds for \( x = 0 \) (it’s actually equality in this case). Again let \( x \in \mathbb{R} \setminus 0 \) arbitrary and apply the Mean Value Theorem for \( f \) on the interval between 0 and \( x \). There exists \( y \) such that \( f(x) - f(0) = f'(y)(x - 0) \), and given the hypothesis in this case implies
\[
|f(x) - 1| = |f'(y)||x| \leq |x|.
\]
But by triangle inequality \(|f(x)| - 1 \leq |f(x) - 1|\), so combining these we get \(|f(x)| \leq |x| + 1, \forall x \in \mathbb{R} \). □
(c) Let $a < b$ arbitrary. By the Mean Value Theorem for $f$ on the interval $[a, b]$, there exists $c \in (a, b)$ such that $f(b) - f(a) = f'(c)(b - a)$. But by assumption $f'(c) \geq 0$, and $b - a > 0$, so it follows that $f(b) - f(a) \geq 0$. □