1. (24 pts) Let $f : X \to Y$ be a function.
(a) (8 pts) Prove that for any subsets $B_1, B_2$ of $Y$
\[ f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2). \]

(b) (8 pts) Prove that for any subsets $A_1, A_2$ of $X$
\[ f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2). \]

(c) (8 pts) Give a concrete example to show that inclusion in part (b) is not, in general, an equality.

2. (21 pts) Suppose that $X$ is an infinite set (possibly non-countable) and $A$ is a countable subset of $X$. Show that if $X - A$ is infinite, then $X$ and $X - A$ have the same cardinality.