1. Given a process of the form
\[ dX = aXdt + \sigma(X,t)dZ \] (where \( a \) is constant),
show that, no matter how complicated the function \( \sigma(.) \) may be, we always have:
\[ E[X_T \mid X_t] = X_t e^{a(T-t)} \]

2. An index (such as the S&P 500) yielding an instantaneous dividend yield \( \delta \) must satisfy (by Feynman-Kac) the following equation:
\[ S_t = E^Q \left[ S_T \exp \left\{ -\int_t^T r_u du \right\} + \int_t^T \exp \left\{ -\int_t^\tau r_u du \right\} \delta, S_\tau d\tau \mid S_t \right] \]
Assume that \( r \) and \( \delta \) are constant. Use the risk-neutral dynamics of such an index and the result from question 1 to verify that the expression on the right-hand side does indeed collapse to \( S_t \).

3. For a European call option on a non-dividend paying stock, show that
\[ \frac{\partial^2 C}{\partial K^2} = e^{-r(T-t)} \rho^Q (K \mid S_t = S) \]