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Estimation of Adult Skeletal Age-at-Death Using the Sugeno Fuzzy Integral

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ABSTRACT Age-at-death estimation of an individual skeleton is important to forensic and biological anthropologists for identification and demographic analysis, but it has been shown that the current aging methods are often unreliable because of skeletal variation and taphonomic factors. Multifactorial methods have been shown to produce better results when determining age-at-death than single indicator methods. However, multifactorial methods are difficult to apply to single or poorly preserved skeletons, and they rarely provide the investigator with information about the reliability of the estimate. The goal of this research is to examine the validity of the Sugeno fuzzy integral as a multifactorial method for modeling age-at-death of an individual skeleton. This approach is novel because it produces an informed decision of age-at-death utilizing multiple age indicators while also taking into consideration the accuracies of the methods and the condition of the bone being examined. Additionally, the Sugeno fuzzy integral does not require the use of a population and it qualitatively produces easily interpreted graphical results. Examples are presented applying three commonly used aging methods on a known-age skeletal sample from the Terry Anatomical Collection. This method produces results that are more accurate and with smaller intervals than single indicator methods. Am J Phys Anthropol 000:000–000, 2009. © 2009 Wiley-Liss, Inc.

Accurate age-at-death prediction is vital for the description and analysis of skeletal remains by biological anthropologists, especially in a medicolegal setting (Lovejoy et al., 1985b). Forensic investigators rely on the accuracy of age indicators for personal identification purposes, whereas biological anthropologists generally have a broader area of applicability for accurate age estimation, such as paleodemographic studies (Schmitt et al., 2002). Although accurate age estimation from skeletal remains is essential, it has been shown that current aging methods are subject to skeletal variation that reduces accurate estimation of age-at-death (Schmitt et al., 2002). As a result, it is necessary to explore different ways to account for this inherent inaccuracy in the aging methods when determining age.

Most of the skeletal aging methods are based on collections with known aged individuals, such as the Terry Anatomical Collection (Trotter, 1981; Hunt and Albanese, 2005). This collection contains mainly black and white adult skeletons with dates of birth ranging from 1822 to 1943. However, populations frequently vary in the timing of their progression through age indicators, especially ones based on stages (Hoppa, 2000; Schmitt et al., 2002). Other factors (e.g., age, genetic variation, environmental factors, diet, disease, activity levels, and taphonomic processes) also affect the accuracy and precision of skeletal age-at-death determination (Meindl et al., 1985; Hoppa, 2000; Boldsen et al., 2002; Hoppa and Vaupel, 2002). Because of the progressive development of bones, skeletally immature and young adult individuals are likely to be aged more precisely than older adult individuals. The skeletal developments of subadults are better documented and include factors such as tooth development and eruption and epiphyseal closure. Adult bones are in a continual process of change as well, but these changes are usually degenerative and influenced by the habitual activities and health of the individual as well as the age of the individual.

Another factor that contributes to uncertainty with age estimation is the age class categories or age ranges used to subdivide age in each method. Skeletal age assessment does not provide an exact age, such as 20 years, but is instead broken up into age classes or ranges, such as 20–25 years or 30–40 years. Segmenting such a continuum into intervals leads to imprecision (Konigsberg et al., 2008). In addition, age ranges are not consistent with the number of years included. There is some inclusion and exclusion of numbers with each range or it may include a 10-year age range for younger individuals and a 50+ years age range for older individuals. This is apparent within an aging method as well as between aging methods. This inconsistent segmenting is a cause for non-crisp boundaries and, therefore, a specific single age based on these methods cannot be reached. The variation present in aging techniques for middle aged or older adults does not necessarily indicate that the techniques need to be perfected as it reinforces

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TABLE 1. Terry collection sample categorized by sex and age group

	Pubic symphysis			Auricular surface			Cranial suture		
Age group (years)	Male	Female	Total	Male	Female	Total	Male	Female	Total
0–9	0	0	0	0	0	0	0	0	0
10–19	4	4	8	5	7	12	7	7	14
20–29	67	44	111	71	48	119	70	49	119
30–39	86	54	140	86	55	141	87	55	142
40–49	83	56	139	81	60	141	84	59	143
50–59	80	60	140	81	66	147	80	67	147
60–69	51	42	93	51	46	97	50	45	95
70–79	28	29	57	27	32	59	30	31	61
80–89	21	30	51	21	32	53	22	33	55
90–99	2	11	13	2	13	15	2	13	15
100–109	0	2	2	0	2	2	0	2	2

TABLE 2. Morphological stages and age ranges for pubic symphysis, auricular surface, and cranial suture closure

Pubic symphysis[a]		Auricular surface[b]		Ectocranial lateral-anterior suture closure[c]		Ectocranial vault suture closure[c]	
Stage	Age range (years)	Stage	Age range (years)	Composite score	Age range (years)	Composite score	Age range (years)
1	18–19	1	20–24	0	<50	0	<49
2	20–21	2	25–29	1	19–48	1,2	18–45
3	22–24	3	30–34	2	25–49	3,4,5,6	22–48
4	25–26	4	35–39	3,4,5	23–68	7,8,9,10,11	24–60
5	27–30	5	40–44	6	23–63	12,13,14,15	24–75
6	30–35	6	45–49	7,8	32–65	16,17,18	30–71
7	35–39	7	50–59	9,10	33–76	19,20	23–76
8	39–44	8	60+	11,12,13,14	34–68	21	Closed (40−)
9	45–50			15	Closed		
10	50+						

[a] Todd 1920.
[b] Lovejoy et al. 1985b.
[c] Meindl and Lovejoy 1985.

that aging is an inconsistent process and the variation in the rates of aging increases over a life span (Klepinger, 2006).

A novel approach to account for inaccuracy in aging methods is to use the fuzzy integral to produce a confidence in skeletal age-at-death. Herein, the term confidence is not a probabilistic concept but instead it is related to fuzzy set theory and can be interpreted in the context of the fuzzy integral as the support or strength regarding an age-at-death. The fuzzy integral is a multifactorial way to analyze skeletal age-at-death that takes into account as much information as possible to reach a decision about a hypothesis. Other multifactorial aging methods (e.g., Acsádi and Nemeskéri, 1970; Lovejoy et al., 1985a; Boldsen et al., 2002) cannot account for as much information as the fuzzy integral. In the examples used in this article, the fuzzy integral takes into account the variable accuracies associated with each method along with the weathering or quality of the skeletal element. However, the fuzzy integral allows investigators to account for any variable that can be quantified. In addition, the goal of using the fuzzy integral is not to model a population or to produce likelihood estimates, but rather to produce results for assessing the age of an individual skeleton without the use of any knowledge about the population from which the method was developed. Finally, the fuzzy integral is not rigid and can easily be structured to meet the needs of individual researchers. Nearly any age-at-death estimation method can be incorporated and numerous variables known or observed about a particular skeleton can be taken into account.

MATERIALS AND METHODS

Data source

The morphological age data used here was collected as a part of another study by Lyle Konigsberg, Nicholas Herrmann, and Daniel Wescott and made available by Lyle Konigsberg (http://konig.la.utk.edu/paleod.htm, 2009). Skeletal remains of known age from the Terry Collection were scored for pubic symphysis morphology (Todd, 1920), auricular surface morphology (Lovejoy et al., 1985b), and ectocranial suture closure (Meindl and Lovejoy, 1985). Sample sizes by sex and decade of life are shown in Table 1. There are more males than females, with the greatest number of individuals being middle to old age adults. Each of these aging methods has an age range associated with each phase (Table 2), which are used in the Sugeno integral.

T1

T2

Fuzzy integral

Fuzzy set theory provides a powerful framework in which to model the uncertainty present in age determination. Fuzzy integrals, while new to anthropological investigations, have been used for hand writing recognition (Gader et al., 1996), computer vision (Tahani and Keller, 1990), face recognition (Keun-Chang and Pedrycz, 2004), machinery fault diagnosis (Xiaofeng et al., 2006), and linguistic fuzzy integrals (Auephanwiriyakul and Keller, 2001). Because age determination methods are not intended to be a rigid set of typological standards but rather describe modal age changes, uncertainty is inherent in skeletal age determination (Meindl et al., 1985). In addition depending on the preservation and quality of the traits, each skeleton has its own degree of error or precision (Boldsen et al., 2002), which can be incorporated into the fuzzy integral model.

When there is a complex decision to be made, many times there are multiple sources of information available. Fuzzy measures produce a value of "worth" for each subset of information sources relative to answering a particular question. For example, when a leader of a country is faced with a complex decision, she relies on her advisors. She takes into account what the advisors or "information sources" are telling her and how much trust she has in each. That is, each "information source" provides a confidence in the goodness of a particular decision, and the leader can fuse those confidences with their value. When each advisor is independent, a weighted average may be appropriate. Conversely, if certain groups of advisors are strongly confident in their decisions, their combined worth can be greater than just the sum of the individual people. For example, the leader may judge that advisors A and B both have an individual worth of 0.3. If advisors A and B agree on a decision, however, the leader may judge their combined advice as worth 1.0, independent of what the others say. This is clearly not probabilistic because of the additivity property of probability theory. Fuzzy measures can be used to produce a value of "worth" for each subset of information sources, and each information source will provide a confidence for a hypothesis. That is, the fuzzy integral is a mechanism to fuse these two quite different types of evidence.

There are two fuzzy integrals: the Sugeno integral (Sugeno, 1977) and the Choquet integral (Murofushi and Sugeno, 1991). The Sugeno integral is used in this article because its internals are easier to conceptualize. Additionally, there are no perceived advantages in using the Choquet over the Sugeno integral for the approach taken in this article. The differentiable form of the Choquet integral is not needed because no procedures are used here for learning the algorithm parameters and no optimizations are attempted through minimization or maximization by differentiation. The Sugeno integral is defined with respect to a fuzzy measure (Wang and Klir, 1992). A fuzzy measure, g, is a real valued function defined on the power set of X (the universe of discourse), 2^X, with range [0,1], satisfying the following properties. Let A and B be two subsets from X.

1. Boundary conditions
   $g(X) = 1$ and $g(\phi) = 0$, where ϕ is the empty set
2. Monotonicity
   $g(A) \leq g(B)$ if $A \subseteq B$
3. Continuity
   If $\{A_i\}$ is an increasing subsequence of subsets of X, then

$$g(\bigcup_{i=1}^{\infty} A_i) = \lim_{i\to\infty} g(A_i)$$

Property 3 is not applicable when X is a finite set, as it is for this application. A fuzzy measure specifies the opinion of the "worth" or "goodness" of each subset of information sources in evaluating a particular hypothesis. Each information source gives a belief or confidence in the hypothesis and the measure lets you know how to weight that belief or confidence. A fuzzy measure g is called a Sugeno λ-fuzzy measure if it additionally satisfies the following property.

4. For all $A, B \subseteq X$, with $A \cup B = \phi$

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B)$$

for some $\lambda > -1$

If λ equals zero, then rule 4 shows that g is a probability measure. For a fuzzy measure g, let $g^i = g(\{x_i\})$. The mapping $x_i \to g^{x_i}$ is called a fuzzy density function. If $X = \{x_1,\ldots,x_N\}$, we sometimes write g^i to mean g^{x_i}. The fuzzy density value, g^i, is interpreted as the (possibly subjective) importance of the single information source x_i in determining the evaluation of a hypothesis. If g is a Sugeno λ-fuzzy measure (Sugeno 1974), then only the densities must be provided, where λ is found by solving for

$$\lambda + 1 = \prod_{i=1}^{N} \left(1 + \lambda g^i\right).$$

When the total number of subsets is relatively small, the g function can be specified where the provided values satisfy properties 1–3 earlier. Depending on the selection of g, many familiar statistics can be recovered, such as the average, median, and even an ordered weighted average (Keller et al., 1994).

Once λ is found, assuming the use of the Sugeno λ fuzzy measure, the Sugeno integral can be calculated. The Sugeno integral, in its discrete form, is defined as:

$$\oint h \circ g = \bigvee_{i=1}^{N} \left(h(x_{(i)}) \wedge g(\{x_{(1)}, \ldots, x_{(i)}\})\right)$$

where x_i is the ith information source (age indicator), $h(x_i)$ is the support of the hypothesis from the standpoint of x_i (the strength in the hypothesis that the skeleton is a particular age according to a single source), N is the number of age indicators used, and $x_{(i)}$ is the ith sorted source, which are sorted in decreasing order according to the $h(x_i)$ values. The $h(x_i)$ values are a function of the quality, a value between 0 and 1, and if the aging method indicates that the age being tested falls within the age interval, either a 0 for false or 1 for true. Formally, it is the minimum of these two numbers. The ^ in the Sugeno integral is a t-conorm and the ^ is a t-norm (Zadeh, 1965). These are generally picked to be the maximum and minimum, respectively, for the Sugeno integral.

Several methods for combining evidence produced by multiple sources include Bayesian reasoning, Dempster-Shafer belief theory, and fuzzy logic. The fuzzy integral differs from the prior in that objective evidence from the individual sources, the experts, and the expected worth of subsets of these sources is considered in the fusion procedure. The result of the fuzzy integral is the support in the hypothesis being tested. The term support should not be confused with the probabilistic concept of the natural log of a likelihood ratio. Support, as used here, is

TABLE 3. List of commonly used coefficients of correlation between age indicators and age-at-death

Indicator	Female	Male	Both sexes	Reference
Endocranial sutures	0.35	0.51	–	Ascádi and Nemeskéri (1970)
Ectocranial sutures	0.34	0.59	0.56	Meindl and Lovejoy (1985)
Ectocranial sutures	–	–	0.57 (L)[a] 0.5 (V)[b]	Meindl and Lovejoy (1985)
Sternal rib ends	–	–	0.75	Dudar et al. (1993)
Proximal humerus	0.34	0.44	–	Ascádi and Nemeskéri (1970)
Proximal femur	0.58	0.56	–	Ascádi and Nemeskéri (1970)
Pubic symphysis	0.64	0.57	0.57	Meindl et al. (1985)
Auricular surface	–	–	0.72	Lovejoy et al. (1985b)
Dental wear	–	–	0.7	Lovejoy (1985)

[a] Lateral.
[b] Vault.

the strength of the hypothesis acquired by fusing these two distinct different sources of information. One interpretation of the Sugeno integral is that it is the search for the maximum grade of agreement between the objective evidence, what the sources (experts) are saying, and the expectations, assigned worth given to subset of sources (Tahani and Keller, 1990).

Accuracy and quality indices

There is a large amount of information that can be unknown about a skeleton in an archaeological or forensic setting, such as the age-at-death of the individual, the amount of exposure to various unknown elements, and cause and timing of death. Therefore, it is important to include as much known or observable information as possible about the skeleton so that all factors available can be taken into consideration when determining skeletal age-at-death. The accuracy of the methods and condition or quality of the skeleton becomes important because they are related to the overall assessment of the skeleton.

Accuracy index. Accuracy in age estimation is defined as how well the estimated or biological age conforms to the real chronological age, and relies on how strongly correlated the biological and chronological ages are to each other. The accuracy value used in this article is the correlation coefficient of the chronological and biological age indicator that is associated with each aging method (Katz and Suchey, 1985; Meindl and Lovejoy, 1985; Meindl et al., 1985). There are biases with using correlation coefficients; such as the use of overlapping phases with intervals containing variable lengths and sample biases because of the differences in when individuals enter and exit stages, but it is a commonly used notion of accuracy with respect to the aging methods (Kemkes-Grottenthaler, 2002). Furthermore, the correlations are used here simply because they are understood by most biological anthropologists and readily available. The correlation coefficient, when considering the relationship between chronological and biological age, can range from 0 to 1, with accuracy increasing as the correlation coefficient approaches one. The correlation coefficient can range from -1 to 1, but if there is ever a negative correlation the accuracy is set to 0. The correlation coefficient value is usually published with the descriptions of the methods or in articles examining the validity of the methods. Males and females differ in their relationship between biological and chronological age, and therefore the correlation coefficients can be valued differently for each sex, but this is not always the case. Population differences in the relationship between biological and chronological age could also be a potential problem but were not considered here. Table 3 provides a list of some commonly used age indicators and their correlation coefficients. T3

Quality. Bone quality is an additional piece of information that can be taken into consideration when analyzing a skeleton. Because the condition of the skeleton is dependent on environmental factors, the quality of a skeleton affects determining age-at-death. A metric is required to describe or determine these variations seen on the bony elements under consideration. In this article, bone quality was evaluated and scored using a modified version of Behrensmeyer (1978) weathering stages provided in *Standards* (Buikstra and Ubelaker, 1994), but with a stage 6 added to take into account the absence of an age indicator. The weathering stages describe the bone's surface texture that "may be altered by heat, plant roots, worms, soil/sediment characteristics, scavengers, and human activity" (Buikstra and Ubelaker, 1994, 97). The phases take into account the taphonomic processes that may have affected the bones of an individual and, in this case, a bone's age indicator. Efremov (1940) described taphonomy as the study of the geological and biological processes that affect organic remains as they pass from the biosphere to the lithosphere. It can also be defined as the postmortem processes which affect "(1) the preservation, observation, or recovery of dead organisms, (2) the reconstruction of their biology or ecology, or (3) the reconstruction of the circumstances of their death" (Haglund and Sorg, 1997, 13). Weathering represents the bones' response to the immediate environment (Ubelaker, 1997). As used here, the weathering stages do not represent the passage of time; rather, they represent the condition of the bone. Systems other than the Behrensmeyer (1978) weathering stages could be used, but it is important to take taphonomy into consideration because it can have differential effects on the bony element under consideration when assessing age.

Quality values are assigned, based on the weathering stages, and are associated specifically with the part of the skeleton for which the aging methods are being applied. If the bone is deteriorated but can still be used, the aging method will still be applied but the decision reached will respectively be associated with a lower support value. The quality indices are ordinal and arbitrarily assigned values. They must be decreasing values and no greater than one and no less than 0. They were assigned in this manner to indicate the least weathering as producing the best results (1) and the most weathering as producing poor or no results (0). A score of "0" is assigned if the bone is not present. For this article, Stage 0 is assigned a quality value of 1.0, Stage 1 is 0.8, Stage 2 is 0.6, Stage 3 is 0.4, Stage 4 is 0.2, Stage 5 is 0.1, and Stage 6 is determined to be a value of 0. However, the data used in this study were collected on museum specimens and therefore the quality index does not vary considerably.

Age-graph construction

The method proposed here is for obtaining skeletal age-at-death. It can be used by biological or forensic anthropologists to improve their analysis of the age-at-death of an adult skeleton. It may also serve as an excellent figure for the forensic anthropologist at trial. One of the main benefits of this approach is that only one skeleton needs to be present to produce the desired results. Age range of the aging method, quality, and accuracy, all of which are collected from a skeleton, can be entered into a chart. Aging methods generally provide age ranges for each phase.

Once the age, accuracy, and quality indices are collected, a hypothesis is tested. The hypothesis is "the skeleton is age _____ (a specific age, not range)". The Sugeno integral, as defined earlier, is repeatedly applied once for each possible age using the accuracy, range, and quality information. Every age, in discrete 1 year increments from 1 to 110, is tested and the age indicators provide input based on if the age being tested is in their respective interval range. Although infants and juveniles are not included in the test sample, the proposed approach can be applied to them without any modification of the technique by using the appropriate aging methods. Therefore, the best known or reliable aging methods can be used when they are most applicable, depending on if the skeleton is an adult, infant, or juvenile. The result of the Sugeno integral, one for each individual age tested, is the maximum of the minimums of g and h at each x-axis unit. The idea behind the minimum calculations are that they are a pessimistic operation, where h is the confidence in the decision from the ith sorted source and g is how much confidence we have in a subset of sources (the first sorted source to the ith) therefore, there should only be as much confidence in the decision as the belief in the sources. If g is less than h, there is less confidence in the decision than what the source is telling us. If g is greater than or equal to h, then we are confident in what the source is telling us. The best pessimistic agreement, what the Sugeno integral produces, is shown by the dashed circle in Figure 1. F1

The Sugeno integral fuses together h and g, where g is accuracy (confidences in subsets of sources) and h is what the sources are telling us, which is the minimum of the quality index value and the {0,1} age test, which tests if the age indicator says that the skeleton is the age being tested. The following is a synthetic example of how the Sugeno integral is calculated. The hypothesis being tested is that the skeleton is of age 28. For this example, the three arbitrary aging methods used indicate that the skeleton could be of age 28, but different bone weathering values are present.

1. First, the h and g values are reported and λ is calculated using the g values.
   $h(x_1) = 0.6 \quad g(x_1) = 0.6$
   $h(x_2) = 0.9 \quad g(x_2) = 0.7$
   $h(x_3) = 0.2 \quad g(x_3) = 0.3$
   $\lambda = -0.8543$
2. Next, sort the sources in decreasing order according to their h values.
   $h(x_{(1)}) = h(x_2) = 0.9$
   $h(x_{(2)}) = h(x_1) = 0.6$
   $h(x_{(3)}) = h(x_3) = 0.2$
3. Calculate the required g subset values.
   g calculations: $g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B)$
   $g(x_{(1)}) = 0.7$
   $g(x_{(1)}, x_{(2)}) = (0.7 + 0.6 + (-0.8543 \times 0.7 \times 0.6)) = 0.9412$
   $g(\{x_{(1)}, x_{(2)}\}, x_{(3)}) = (0.9412 + 0.3 + (-0.8543 \times 0.9412 \times 0.3)) \approx 1$
4. Calculate the Sugeno integral. From $i = 1$ to N, take the minimum of the h value of the ith sorted source and the g value of the first sorted source to the ith sorted source (this is a subset). Take the maximum of the N values.
   Sugeno integral $= \max(\min(h(x_{(1)}), g(x_{(1)})), \min(h(x_{(2)}), g(x_{(1)}, x_{(2)})), \min(h(x_{(3)}), g(\{x_{(1)}, x_{(2)}\}, x_{(3)}))) = \max(\min(0.9, 0.7), \min(0.6, 0.9412), \min(0.2, 1))$
   Maximum of the minimums $= (0.7, 0.6, 0.2) = 0.7$

For this example, the best pessimistic agreement, or the support in the hypothesis, is 0.7 for the skeleton aged 28. The support in the hypothesis is the best (maximum) pessimistic (minimum) agreement. This process is repeated and each age from 1 to 110 is tested.

Age-graph categories

Once all of the possible individual age hypotheses have been tested, the numbers produced by the Sugeno integral are used to form a graph that simplifies the subsequent analysis phase. As illustrated in Figure 2, at least four different potential graph cases are possible: specific (Fig. 2, Graph A), interval (Fig. 2, Graph B), reconsideration (Fig. 2, Graph C), and inconclusive (Fig. 2, Graph D). F2

A graph is classified into one of the four prior mentioned categories according to the characteristics of its intervals. In this work, an interval is defined to be a set of consecutive ages in which all of the ages have the same support in the hypothesis value. Specifically, the intervals of interest are those that have the maximum

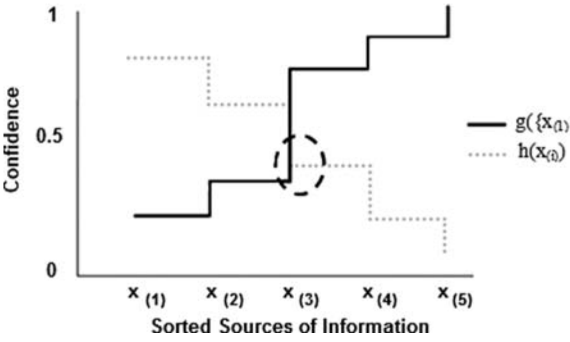
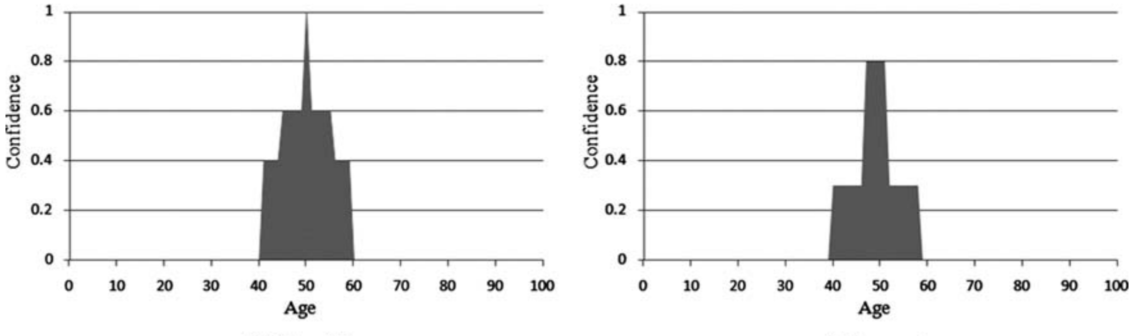
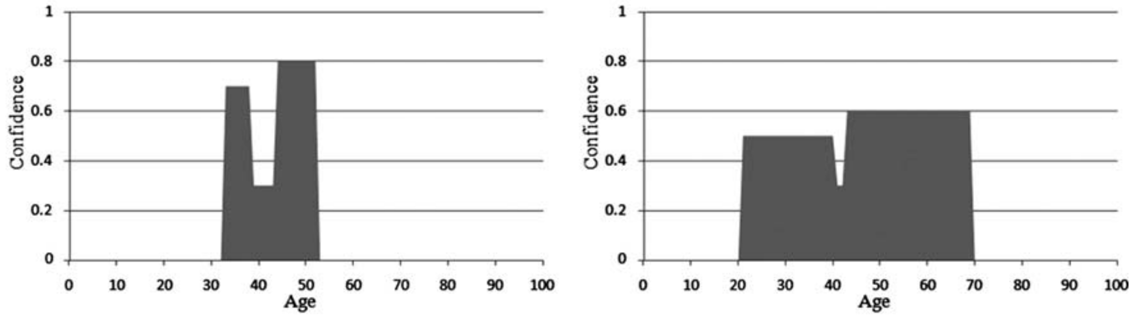


Fig. 1. The best pessimistic agreement between g and h in the Sugeno integral. The dotted circle is the maximum of the minimums of g and h at each x-axis unit.

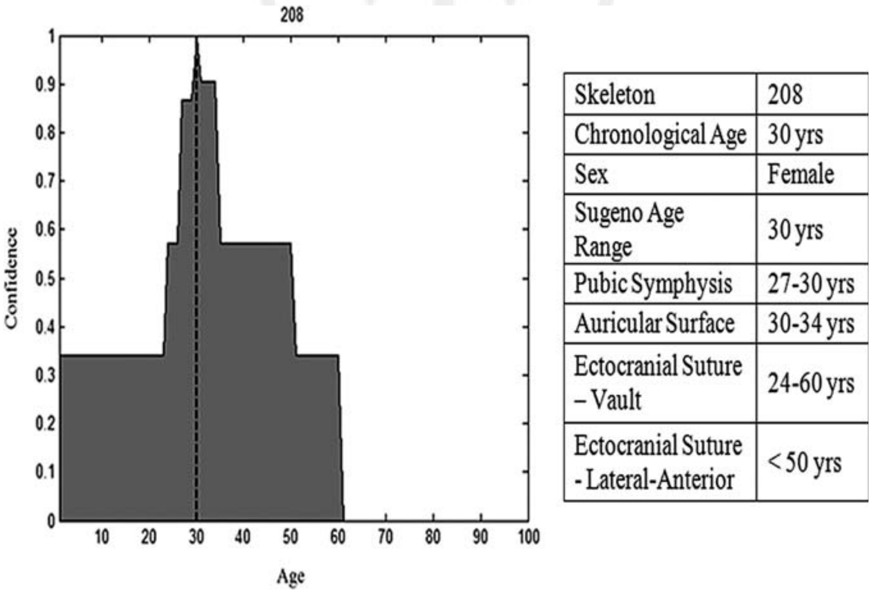


(A) Specific (B) Interval



(C) Reconsideration (D) Inconclusive

Fig. 2. Illustration of potential graph categories for the Sugeno integral results (see text for explanation of each graph).

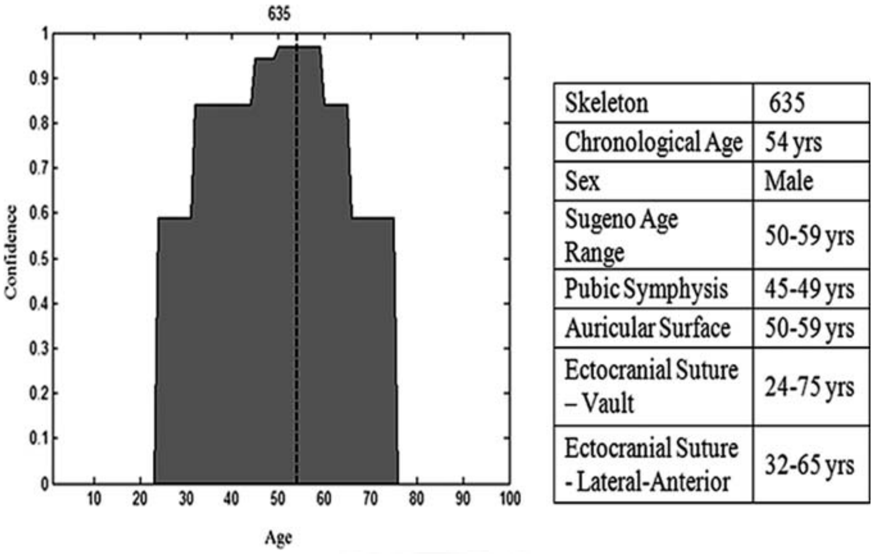


Skeleton	208
Chronological Age	30 yrs
Sex	Female
Sugeno Age Range	30 yrs
Pubic Symphysis	27-30 yrs
Auricular Surface	30-34 yrs
Ectocranial Suture – Vault	24-60 yrs
Ectocranial Suture - Lateral-Anterior	< 50 yrs

Fig. 3. Graph of Sugeno integral age range for Terry Skeleton 208 illustrating a "specific age" type graph. The dashed vertical line in the graph indicates the chronological age and the intervals in the table represent input of age range for each aging method.

width, cardinality, i.e., the value before the left endpoint and the value after the right endpoint have different support values from that of the interval. A plateau interval is defined here to be an interval such that the interval directly before it and the interval directly after it have smaller support in the hypothesis values. The following tests, analyzed in the order specified below, are used to determine the type of graph. Each test is a yes or no question.

- Test 1: Is the length of the maximum plateau interval greater than or equal to 30 years? If the answer is yes, then the graph is placed in the category inconclusive, if not, then the next test is conducted.
- Test 2: Is there a single plateau interval and is its length equal to one? If yes, the result is specific age.
- Test 3: Is there a single plateau interval and is its length 29 or less years? If yes, the result is age interval.



Skeleton	635
Chronological Age	54 yrs
Sex	Male
Sugeno Age Range	50-59 yrs
Pubic Symphysis	45-49 yrs
Auricular Surface	50-59 yrs
Ectocranial Suture – Vault	24-75 yrs
Ectocranial Suture - Lateral-Anterior	32-65 yrs

Fig. 4. Graph of Sugeno integral age range for Terry Skeleton 635 illustrating an "interval" type graph. The dashed vertical line in the graph indicates the chronological age and the intervals in the table represent input of age range for each aging method.

In the case that the graph does not pass one of the above three tests then it is labeled as type reconsideration.

The proposed four categories are based on observable characteristics and they model the different desires to interpret applicable results. Technically, there are only two fundamental categories based on intervals. Either there is a single plateau interval, graph types specific and interval, or there are multiple plateau intervals, such as type reconsideration. Type inconclusive can fit into either of these two categories. Type specific is included to capture the ideal age-at-death classification scenario and type inconclusive expresses the notion that in some cases the maximum plateau interval is too large to make any valuable assessment. The 30 years long plateau used in Test 1 is used here for illustrative purposes. The investigators can modify the length of the plateau to meet their specific needs. Other types of graphs, which increase the degree of specificity for these two main interval-based categories, can be modeled if desired.

Program and output

The implementation of the fuzzy integral and all graphs produced are generated in MATLAB, which is a technical computing language and interactive environment that allows for data visualization, analysis, and numeric computation (http://www.mathworks.com/products/matlab/description1.html, 2007). MATLAB is used in many domains such as image processing, pattern recognition, and bioinformatics.

The type of graph, the support in the hypothesis value for the maximum plateau interval, and a sorted list of all intervals and the ages included in them are provided. The reason for reporting the maximum plateau interval is to let the researcher know where the greatest strength is regarding the age-of-death. In the case of reconsideration, a sorted list, in decreasing order, is produced from the top *k* intervals. A *k* of 2 is chosen here; therefore the top two intervals are reported. Two was selected because with this data set there are only one or two maximum intervals present.

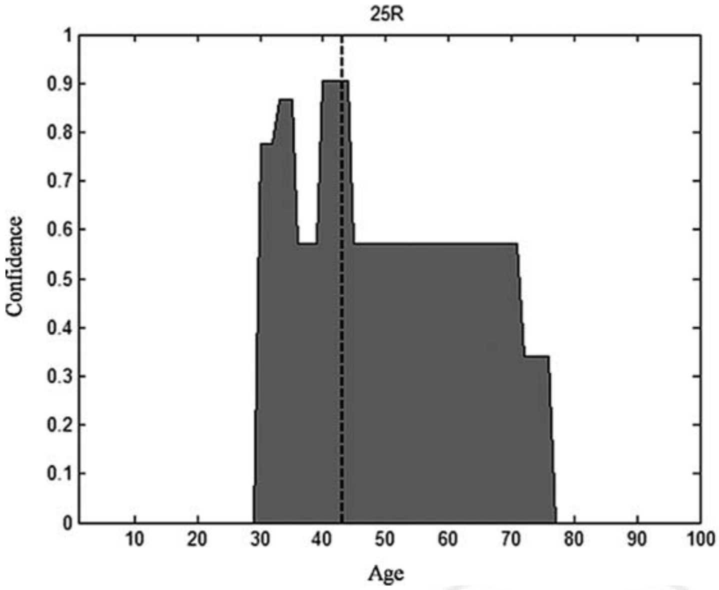
RESULTS AND DISCUSSION

Examples of selected graphs from Terry collection skeletons

The aging methods, skeletal quality, and accuracy indices are used by the Sugeno integral to produce a confidence that the skeleton is an age (best pessimistic agreement), which are assembled together to produce graphical results. These graphs are classified, by a computer program, according to specific, interval, reconsideration, and inconclusive. The accuracy values for this data set are the correlation coefficients that are published with or about the aging methods. Because the Terry Collection contains skeletons that are from a museum collection and are in good condition, a quality of 1.0 was assigned to all skeletons. From the data set, 735 skeletons (412 male and 323 female) have all age indicators present and produced classifiable graphs. As mentioned, there are four possible graph types. Figures 3–6 are some examples from the data collected from the Terry Collection for specific age, interval, reconsideration, and inconclusive. Terry skeleton 208 (see Fig. 3) F3 provides an example of a "specific age" type graph where the chronological and biological ages are in agreement. Figure 4 illustrates an 'interval" type graph where the F4 chronological age of Terry 635 falls within the interval with the greatest support. The graph of the Sugeno interval for Terry 25R (see Fig. 5) illustrates and example F5 of a "reconsideration" type graph. Here there are two peaks indicating two age ranges. Finally, the Sugeno interval for Terry 779R (see Fig. 6) illustrates an "inconclusive" F6 type graph where there is little support for any reasonable age range.

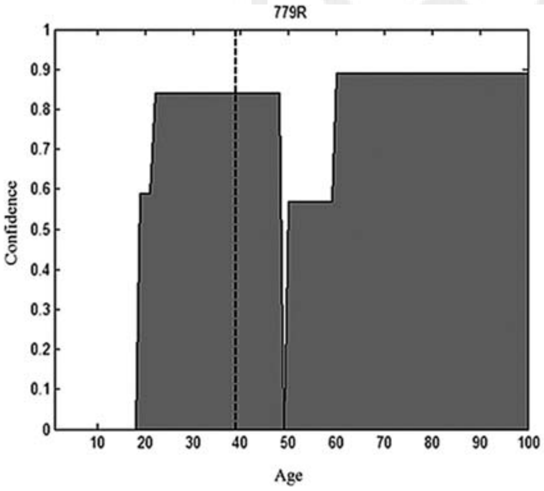
Theoretical range of the Sugeno fuzzy integral

Because of the preservation and currently available information collected from the Terry Collection, the range



Skeleton	25R
Chronological Age	43 yrs
Sex	Female
Sugeno Age Range 1	40-44 yrs
Sugeno Age Range 2	33-35 yrs
Pubic Symphysis	30-35 yrs
Auricular Surface	40-44 yrs
Ectocranial Suture – Vault	30-71 yrs
Ectocranial Suture - Lateral-Anterior	33-76 yrs

Fig. 5. Graph of Sugeno integral age ranges for Terry Skeleton 25R illustrating a "reconsideration" type graph. The vertical line in the graph indicates the chronological age, whereas the two peaks indicate potential age ranges 1 and 2. The intervals in the table represent input of age range for each aging method.



Skeleton	779R
Chronological Age	39 yrs
Sex	Male
Sugeno Age Range 1	60-100 yrs
Sugeno Age Range 2	22-48 yrs
Pubic Symphysis	50+ yrs
Auricular Surface	60+ yrs
Ectocranial Suture – Vault	22-48 yrs
Ectocranial Suture - Lateral-Anterior	19-48 yrs

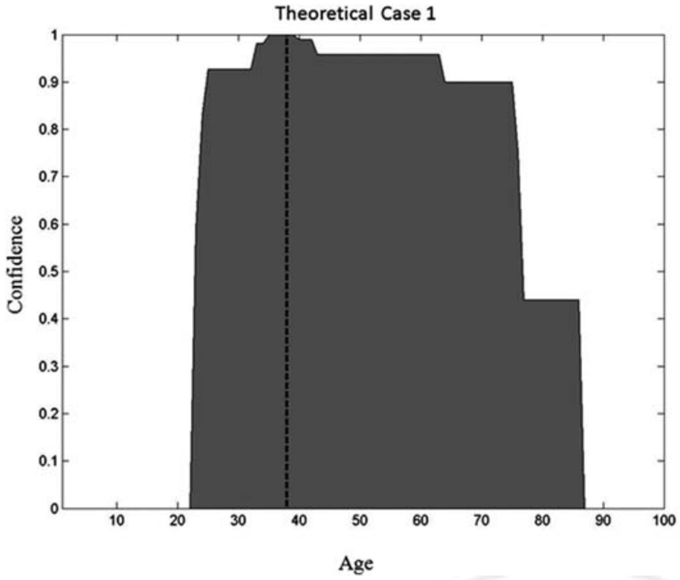
Fig. 6. Graph of Sugeno integral age ranges for Terry Skeleton 779R illustrating an "inconclusive" type graph. The dashed vertical line in the graph indicates the chronological age, whereas the two peaks indicate potential age ranges 1 and 2. The intervals in the table represent input of age range for each of the aging methods.

of the Sugeno integral cannot be fully understood for different situations of age-at-death determination of a skeleton. Because the data collected did not contain information for more than three age indicators nor varying skeletal quality, the following theoretical range scenarios are provided to demonstrate the cases that are not present in the collected data. The scenarios are (1) a skeleton with all methods in agreement on a specific age with a quality of 1.0, (2) the same example as Case 1 with varying qualities, (3) one skeleton from the existing data set has its qualities varied, and (4) a skeleton with disagreement among all of the aging methods. These cases demonstrate the range of the Sugeno integral for this specific problem of determining skeletal age-at-death. The accuracy indices reported in Table 3 are used in the theoretical scenarios.

Case 1: All methods in agreement on a specific age with a quality of 1.0

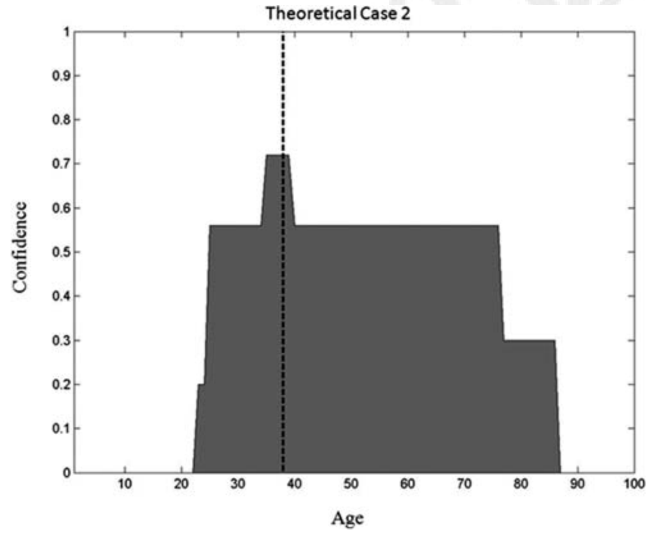
A theoretical 38-year-old male with a quality of 1.0 and data for eight aging methods (pubic symphysis, auricular surface, ectocranial suture closure, endocranial suture closure, sternal rib end morphology, proximal humerus, and proximal femur) that are in agreement is illustrated the first scenario (see Fig. 7). The Sugeno interval includes the chronological age and has an age range where the highest confidence is 37–39 years. This

F7



Aging Method	Age Range
Pubic Symphysis	35-39
Auricular Surface	35-39
Ectocranial Sutures – vault	24-75
Ectocranial Sutures- lateral	23-63
Sternal Rib Ends	33-42
Endocranial Sutures	35-39
Proximal Humerus	37-86
Proximal Femur	25-76

Fig. 7. Sugeno interval graph of Case 1 with aging method inputs. The dashed vertical line indicates chronological age and the peak indicates the maximum Sugeno age range. The resulting maximum plateau interval is 37–39 years.



Aging Method	Quality	Age Range
Pubic Symphysis	0.6	35-39
Auricular Surface	0.8	35-39
Ectocranial Sutures	0.2	24-75
Sternal Rib Ends – vault	0.5	23-63
Sternal Rib Ends – lateral	0.5	33-42
Endocranial Sutures	0.4	35-39
Proximal Humerus	0.3	37-86
Proximal Femur	0.7	25-76

Fig. 8. Result graph of Case 2 with aging method inputs. The dashed vertical line indicates chronological age and the peak is the maximum Sugeno integral plateau interval. The resulting maximum plateau interval is 35–39 years.

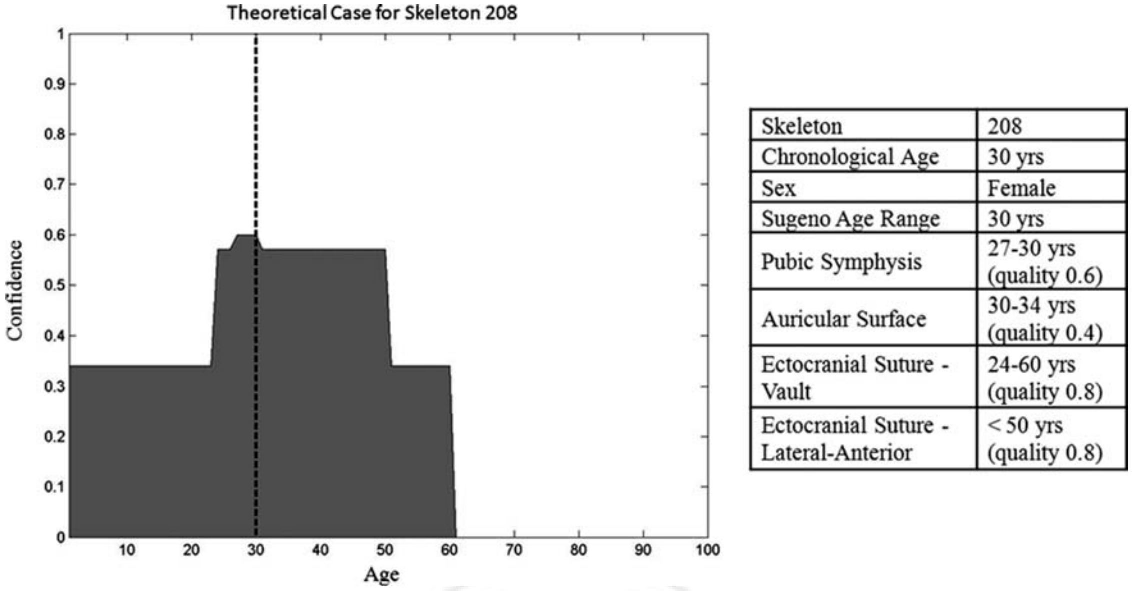
case represents the best possible scenario because of the skeleton being of the highest quality and all of the methods being in agreement. The results produce an age range that includes the age-at-death.

Case 2: All methods in agreement on a specific age with varying bone quality

Theoretical Case 2 has the same input or age ranges as Case 1 (see Fig. 7) but the qualities are varied (see Fig. 8). These qualities were chosen at random. The interval 35–39 years has the highest confidence and still contains the 37–39 years interval from Case 1. However, the age range is less precise now. The age-at-death of the skeleton can still be determined although the skeletal quality is low for certain age indicators. F8

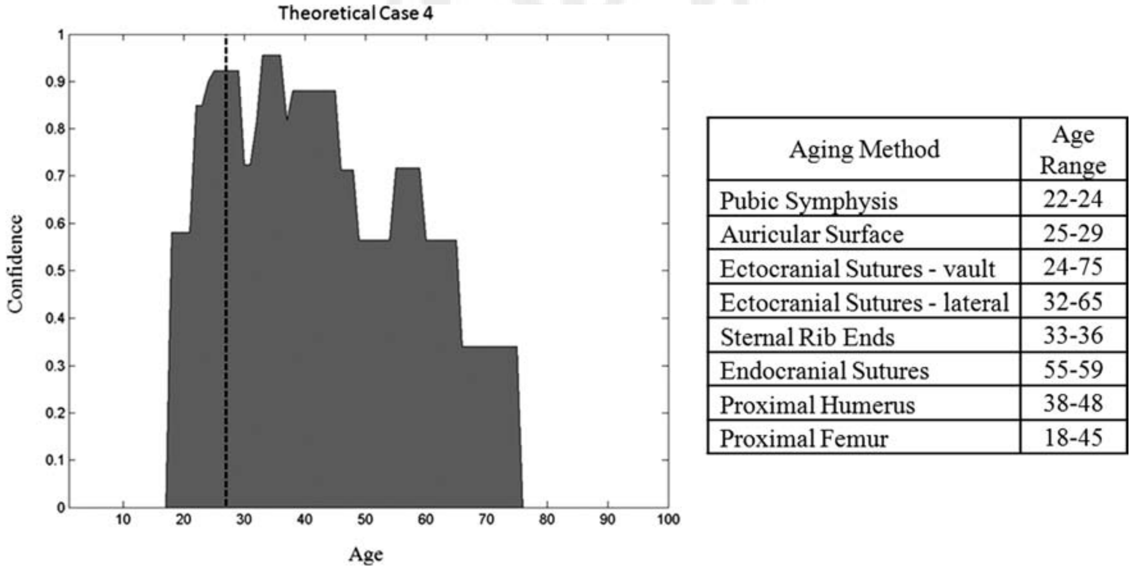
Case 3: One skeleton from the existing data set has its qualities varied

To further demonstrate how bone quality could affect the outcome, the bone qualities for one individual are changed from the original value of 1.0 to 0.6 for the pubic symphsis, 0.4 for the auricular surface, and 0.8 for the ectocranial sutures. These quantities are chosen at random for the purpose of this exercise. The results of the original data with a quality value of 1.0 were included in the prior section (see Fig. 3). The chronological age for skeleton 208 is 30 years. The age range found by the Sugeno integral is 27–30 years (see Fig. 9). Although skeleton 208 is classified as a specific age graph with a quality of 1.0, the different qualities produced an interval age that still includes the chronological age. This demonstrates that different F9



Skeleton	208
Chronological Age	30 yrs
Sex	Female
Sugeno Age Range	30 yrs
Pubic Symphysis	27-30 yrs (quality 0.6)
Auricular Surface	30-34 yrs (quality 0.4)
Ectocranial Suture - Vault	24-60 yrs (quality 0.8)
Ectocranial Suture - Lateral-Anterior	< 50 yrs (quality 0.8)

Fig. 9. Results of Case 3 for Terry Skeleton 208 with aging method inputs. The dashed vertical line indicates chronological age. The Sugeno age ranges are indicated by the peak on the graph with a resulting maximum plateau interval of 27–30 years.



Aging Method	Age Range
Pubic Symphysis	22-24
Auricular Surface	25-29
Ectocranial Sutures - vault	24-75
Ectocranial Sutures - lateral	32-65
Sternal Rib Ends	33-36
Endocranial Sutures	55-59
Proximal Humerus	38-48
Proximal Femur	18-45

Fig. 10. Results of Case 4 with aging method inputs. The dashed vertical line indicates chronological age and the multiple peaks are the Sugeno age ranges.

qualities could play an important role for age at death determination.

Case 4: Disagreement among all of the aging methods

A theoretical 27-year-old female with a quality index of 1.0 is used in Case 4 to show the results when all the age methods used are not in agreement. As can be seen F10 from the graph produced (see Fig. 10), there are too many intervals created to make a conclusion about age-at-death. By setting the quality at a 1.0, the importance of the accuracies of the age indicators can be seen. The methods with the greatest accuracies have the largest impact, such as the sternal rib ends with an accuracy of 0.75 producing the maximum plateau interval of 33–36 years. There is some overlap among the age indicators, but they do not agree overall. This is an extreme example, but it demonstrates a case of the inconclusive graph type. When this occurs, it is an indication to the observer that the data needs to be reanalyzed or something is not "typical" about the skeleton.

Distance between chronological and biological age-at-death

A biological age-at-death metric is used to understand how well the graphs correspond with chronological age-at-death. If the true chronological age falls within the maximum Sugeno integral interval, then it is classified as *In Interval*. If it falls outside of this interval, then the absolute value of the difference between the chronologi-

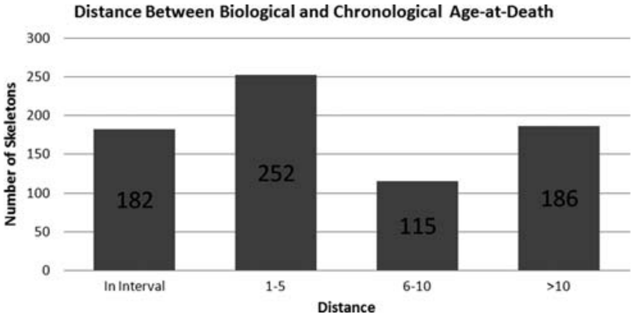


Fig. 11. Distance between biological and chronological age-at-death for the data set.

TABLE 4. Sugeno integral results compared with the individual aging methods with respect to the category In the Interval and the width of the interval

	In the interval		
	Width of interval		
	0–5	6–10	>10
Sugeno results	119	58	5
%	16.00%	8.00%	0.70%
Pubic symphysis	52	37	88
%	7.00%	5%	12%
Auricular surface	87	45	82
%	12%	6%	11%

cal age and the endpoint of the nearest maximum interval is used. If this value is less than or equal to 5 years, it is classified as 1–5, else if it is less than or equal to 10 years it is classified as 6–10, and else if it is greater than 10 years it is labeled >10. As can be seen in Figure F11 11, there are 182 skeletons classified as *In Interval* and 252 classified as 1–5 years. These are considered to be the best categories for classification and together make up 59% of the data. Therefore, over half of the data fell within 5 years of the maximum interval. For the case of reconsideration, the top two intervals are used.

Comparison of Sugeno integral to individual aging methods

The Sugeno integral results are classified into graph categories. Through the use of maximum intervals, it produces smaller intervals containing the chronological age-at-death than the individual aging methods. A single aging method, which is the pubic symphysis, auricular surface, and the ectocranial sutures for this case, is determined to have found the correct biological age-at-death if the chronological age falls into the aging method's interval. The ectocranial sutures are not included in this comparison because all of the age ranges are larger than 10 years and, therefore, they do not produce intervals with a comparable width of acceptable accuracy, e.g., less than 10 years. The following measures how many skeletons' chronological ages fall into the Sugeno integral's maximum interval, or the top two for reconsideration, verses each individual aging method. The width of the interval that the age fell into is recorded and used to classify the results into 0–5, 6–10, and >10 categories. This is a measure of precision. The Sugeno integral produces better results than the single aging methods for T4 all cases except for greater 10 (Table 4). This is desired because it indicates that the Sugeno integral is finding more accurate, smaller interval results.

TABLE 5. Percentage of skeletons that are 0–5, 6–10, and **>***10 years from the maximum intervals and width of that interval*

Distance	0–5 years Width of interval			6–10 years Width of interval			>10 years Width of interval		
Method	0–5	6–10	>10	0–5	6–10	>10	0–5	6–10	>10
Sugeno integral	35%	19%	5%	10%	5%	2%	16%	9%	1%
Pubic symphysis	22%	15%	13%	9%	7%	1%	18%	13%	2%
Auricular surface	30%	14%	14%	10%	4%	2%	14%	9%	3%

The next analysis, Table 5, uses the distance of the T5 chronological age-at-death to the nearest maximum interval endpoint. The results are grouped into the categories of 0–5, 6–10, and >10 and are separated according to the width of the interval. For the category representing the most accurate scenario, 0–5 years, the Sugeno integral has a higher percentage of correctly aged skeletons. This demonstrates that the Sugeno integral finds results where the chronological and biological age-at-death is close to one another and the interval is small, i.e., more precise results. In the case of 6–10 years, the Sugeno integral and individual aging methods produce similar results. Lastly, in the greater than 10 category the individual aging methods classify more skeletons than the Sugeno integral. However, this is the least accurate classification category and because the Sugeno integral was more accurate in the 0–5 years category and was similar in the 6–10 years category, there are less skeletons for the Sugeno integral to classify.

CONCLUSIONS

Multifactorial age methods have been shown to provide more reliable age-at-death estimations than single age estimation methods, and this is the case for the Sugeno interval. Although it is not possible to directly test the accuracy of the Sugeno fuzzy interval to other multifactorial methods, it does have several advantages over other published methods. First, the Sugeno interval can be used for a single skeleton, which is useful especially to forensic anthropologists. Second, the method can be customized to meet the specific needs of the investigator. That is, the age methods can be changed and observable information from the skeleton (quality, sex, and other variables) can be incorporated into the hypothesis test.

Given the difficulty of accurate age-at-death analysis for an individual skeleton, such as the inaccuracies and discrepancies observed among the various aging methods, the results found by using the Sugeno integral are encouraging. The Sugeno integral produces results that are used to answer inquiries such as: what is the most confident age-at-death estimation, what is the most confident age interval, and what type of age-at-death graph is this skeleton based on a pre-defined set of user specified graph types? Here we demonstrated the usefulness of the Sugeno interval as a multifactorial age method using a limited amount of readily available information (three aging methods and a fixed quality of 1.0) gathered from the Terry collection. However, the results presented here are not far from what would be expected when looking at the overlap from the input age ranges from each aging method in the typical forensic anthropological case. The most common discrepancy among the graphs can be attributed to the three methods not finding the same age within 10 years of each other. It is possible that this could be corrected by accounting for biases in the methods used. Finally, we also illustrate several scenarios that demonstrate the theoretical range of the Sugeno integral by using additional age indicators and their accuracies and ranging qualities.

The results reinforce the already known notion that inaccuracies are present in the age indicators to determine age-at-death of a skeleton. Not every age-at-death will be reached with any amount of information because of the inherent flaws in the aging methods, such as over aging younger individuals and under aging older individuals. In general, the classifications of the graphs are encouraging. There are very few inconclusive, with the majority of the graphs being classified as interval. This is promising because of the fact that the inputs of the aging methods are in the form of age ranges/intervals.

As Ubelaker (2000) describes, the "scientific progress in the estimation of age at death involves greater awareness of the variation involved, new techniques available for different structures, and greater appreciation of the importance of regional and temporal variation in the aging process". The Sugeno integral provides the initial framework to take into account the various and varying processes involved with skeletal age-at-death analysis. As a result, the use of the Sugeno integral has significant potential in age-at-death estimations in forensic and bioarchaeological investigations.

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