

## PHY-5115/5116: MATHEMATICAL METHODS I/II (Dr R Fiebig)

The primary text for the courses is:

G B Arfken and H J Weber, Mathematical Methods for Physicists, 6th Edition, Elsevier Academic Press, ISBN 0-12-059876-0.

In PHY-5115 selected material from Chapters 1,2,3,6,7,4 will be covered.

in PHY-5116 selected parts of the remaining Chapters will be covered.

### 1 VECTOR ANALYSIS

- 1.1 Definitions
- 1.2 Rotation of coordinate axes
- 1.3 Scalar product
- 1.4 Vector product
- 1.5 Triple products
- 1.6 Gradient
- 1.7 Divergence
- 1.8 Curl
- 1.11 Gauss' theorem
- 1.12 Stokes' theorem
- 1.15 Dirac delta function

### 2 VECTOR ANALYSIS IN CURVED COORDINATES AND TENSORS

- 2.1 Orthogonal coordinates
- 2.2 Differential vector operators
- 2.5 Spherical polar coordinates
- 2.6 Tensor analysis
- 2.7 Contraction, direct product
- 2.9 Pseudotensors, dual tensors
- 2.10 General tensors

### 3 DETERMINANTS AND MATRICES

- 3.1 Determinants
- 3.2 Matrices
- 3.3 Orthogonal matrices
- 3.4 Hermitean matrices, unitary matrices
- 3.5 Diagonalization of matrices

### 4 GROUP THEORY

- 4.1 - Definition of a group [242]
- 4.7 Discrete groups [291]
- 4.1 - Homomorphism, isomorphism [243]
- 4.7 - Classes and character [293]
- 4.7 - Subgroups and cosets [293]
- 4.1 - Representations - reducible and irreducible [245]
- 4.2 Generators and continuous groups [246]
- 4.2 - Rotation groups SU(2) and SO(3) [250]
- 4.2 - Rotation of functions and orbital angular momentum [251]
- 4.2 - SU(2)-SO(3) homomorphism [252]
- 4.2 - SU(2)-isospin and SU(3)-flavor symmetry [256]
- 4.3 Orbital angular momentum [261]
- 4.3 - Ladder operator approach [262]
- 4.3 - Summary of Lie groups and Lie algebras [264]

4.4	Angular momentum coupling [266]
4.4 -	Clebsch-Gordan coefficients: SU[2]-SO[3] [267]
4.4 -	Spherical tensors [271]
4.4 -	Young tableaux for SU(n) [274]
4.5	Homogeneous Lorentz group [278]
6	FUNCTIONS OF A COMPLEX VARIABLE I
6.1	Complex Algebra
6.2	Cauchy-Riemann Conditions
6.3	Cauchy's Integral Theorem
6.4	Cauchy's Integral Formula
6.5	Laurant Expansion
6.6	Singularities
6.7	Mapping
7	FUNCTIONS OF A COMPLEX VARIABLE II
7.1	Calculus of Residues
8	THE GAMMA FUNCTION
8.1	Definitions, Simple Properties
8.2	Digamma and Polygamma Functions
5.9	Bernoulli Numbers, Euler-Maclaurin Formula [376]
8.3	Sterling's Series
8.4	The Beta Function
8.5	The Incomplete Gamma Functions and Related Functions
9	DIFFERENTIAL EQUATIONS
9.2	First-Order Differential Equations
9.5	Series Solutions, Frobenius' Method
9.6	A Second Solution
10	STURM-LIOUVILLE THEORY - ORTHOGONAL FUNCTIONS
10.1	Self-Adjoint Differential Equations
10.2	Hermitian Operators
10.3	Gram-Schmidt Orthogonalization
10.4	Completeness of Eigenfunctions
10.5	Green's Function – Eigenfunction Expansion
11	BESSEL FUNCTIONS
11.1	Bessel Functions of the First Kind
11.2	Orthogonality
11.3	Neumann Functions, Bessel Functions of the Second Kind
11.4	Hankel Functions
11.5	Modified Bessel Functions
12	LEGENDRE FUNCTIONS
12.1	Generating Function
12.2	Recurrence Relations and Special Properties
12.3	Orthogonality
12.5	Associated Legendre Functions
12.6	Spherical Harmonics
12.8	The Addition Theorem for Spherical Harmonics
12.10	Legendre Functions of the Second Kind

- 13 MORE SPECIAL FUNCTIONS
  - 13.1 Hermite Functions
  - 13.2 Laguerre Functions
  - 13.3 Chebyshev Polynomials
  - 13.4 Hypergeometric Functions
  - 13.5 Confluent Hypergeometric Functions
  
- 14 FOURIER SERIES
  - 14.1 General Properties
  - 14.2 Advantages, Uses of Fourier Series
  - 14.3 Applications of Fourier Series
  - 14.4 Properties of Fourier Series
  - 14.6 Discrete Fourier Transform
  
- 15 INTEGRAL TRANSFORMS
  - 15.1 Integral transforms
  - 15.2 Development of the Fourier Integral
  - 15.3 Fourier Transforms - Inversion Theorem
  - 15.4 Fourier Transform of Derivatives
  - 15.5 Convolution Theorem
  
  - 15.8 Laplace Transforms
  - 15.9 Laplace Transforms of Derivatives
  - 15.10 Other Properties
  - 15.11 Convolution (Faltungen) Theorem
  - 15.12 Inverse Laplace Transform
  
- 16 INTEGRAL EQUATIONS
  - 16.1 Introduction
  - 16.2 Integral Transforms, Generating Functions
  - 16.3 Neumann Series, Separable (Degenerate) Kernels
  - 16.4 Hilbert-Schmidt Theory
  
- 17 CALCULUS OF VARIATIONS
  - 17.8 Rayleigh-Ritz Variational Principle

## EXAMS AND MORE

- Homework will be collected, but not graded. The fractional return on the homework will be assigned a score  $H = 0 \dots 100$ . Homework is due two weeks after assignment (not counting spring break). The deadline for homework submission is at the conclusion of the final exam.
- There will be three exams including the final. The length of each exam is the usual class period. Each exam score  $E_i = 0 \dots 100$ ,  $i = 1, 2, 3$ , carries equal weight. The class textbook only is allowed, and required. Other materials are a calculator, a ruler, paper and pen(cil).
- The overall score  $X$  for the course is determined by combining homework and exams as

$$X = \frac{1}{2}H + \frac{1}{2} \frac{E_1 + E_2 + E_3}{3}.$$

- *Tentative* exam schedule, Fall 2011
  - Oct-04 (Tue)
  - Nov-08 (Tue)
  - Dec-08 (Thu) 12:15-13:45h
- Announcements made in class may supersede syllabus rules!
- An approximate guideline for the grading scale is:

Points $X$	Grade
95–100	$A$
90–95	$A$
85–90	$A-$
80–85	$B+$
75–80	$B$
70–75	$B-$
65–70	$C+$
60–65	$C$
55–60	$C-$
50–55	$D+$
45–50	$D$
40–45	$D-$
0–40	$F$

## HOMEWORK ASSIGNMENTS

- 1.2 ( 13) 1,2
- 1.3 ( 17) 3
- 1.4 ( 22) 7,9
- 1.5 ( 29) 5,7,13
- 1.6 ( 37) 1,3
- 1.7 ( 42) 3,5
- 1.8 ( 47) 7,8,11,14,17
- 1.11 ( 62) 2,3
- 1.12 ( 67) 1,10
- 1.15 ( 91) 1,7,13,23
  
- 2.1 (109) 2,3,6
- 2.2 (113) 3
- 2.5 (128) 1,6,8,16
- 2.6 (138) 2
- 2.7 (140) 2
- 2.9 (149) 3,4,12
  
- 3.1 (174) 1,2,3,6
- 3.2 (187) 1,4,5,7,11,13,15,36
- 3.3 (206) 1,6,9,10
- 3.4 (212) 1,5,12,13,16,18,20,24
- 3.5 (226) 3,6,8,10,19,30,33
- 3.6 (236) 1,5,16,17,18
  
- 4.7 (200) 2,4,5,6,8,9,13,17
- 4.1 (246) 2,3,4
- 4.2 (260) 2,4
- 4.3 (266) 1
- 4.5 (283) 1
  
- 6.1 (409) 8,10,16
- 6.2 (416) 2,5,7
- 6.3 (424) 3
- 6.4 (429) 2,4,8
- 6.5 (437) 3,8,9
- 6.6 (442) 2,4
- 6.7 (450) 1,4,6
  
- 7.1 (474) 1,5,11,14,27