The probability of something happening is a way of representing the likelihood of some occurrence. This is the basis of inference - we have often heard of statements like -

There is a 90% chance of rain today.

There is a 50-50 chance that I will make an A in this class.

The probability of getting a heads when we toss a coin is 1/2.

To really understand probability, we have to look at a few basic definitions:
3.1 - Events, Sample Spaces and Probability

An Experiment is any planned activity that results in exactly one outcome that cannot be predicted with certainty.

Examples include tossing a coin, rolling a die, taking a survey and asking people about their opinion on abortion etc.

Any observation is the result of one run or trial of an experiment. Any experiment has a set of all possible results or outcomes called the sample space, \( S \). Any subset of the sample space is called an event. Usually we denote events by upper case letters from the beginning of the alphabet: \( A, B, C \) etc. We say an event occurs if any outcome in the event occurs.
examples:

1) Roll a die:

   $S =$

   $A =$ get an even number =

   $B =$ get an odd number =

2) Play basketball - # of attempts before you can shoot a basket

   $S =$
3) Put an item on test and measure its lifetime:

\[ S = \]

**Note** a sample space can be finite, discrete infinite or continuous infinite.

Finally, we define something called a **simple event**, which is defined as an event that contains exactly one outcome. These are denoted by \( E \) with a subscript like \( E_1, E_2, E_3 \) . . . etc. Examples...

Roll a 6 when you roll a die, get a head in a toss of a coin.
We talk about probabilities of events. This is denoted by $P(A)$, where $A$ is some event.

So $P(A)$ really is a function which assigns a numerical value to the likelihood of an event.

Two Definitions of Probability

1) The probability of an event is the proportion of time that the event occurs in an infinite number of trials - Relative Frequency Approach
To really understand this concept – think of what it means to get a head when you toss a coin. If someone asks you what is the probability of getting a head on a single toss of a coin, you would say ........1/2 – right?

Well does that mean that if one tosses a coin twice, one should get one head and one tail? ??? How about if we toss a coin four times – 2 heads and 2 tails? ??? So what does it mean when we say that the probability of getting a head in a single toss of a coin is 1/2???

The explanation is given on page 117, look at Figure 3.2
The second approach/definition of probability is explained through what is called the classical approach:

2) Suppose an experiment has $N$ equally likely and mutually exclusive outcomes. Suppose that $s$ of these lead to the occurrence of some event, $A$. Then the probability of the event is

$$P(A) = s/N \quad - \text{Classical Approach}$$

So when we toss a coin once, our sample space consists of two equally likely points \{H, T\}. Thus the probability of getting a heads is $P(H) = \frac{1}{2} = \text{#of H/Total number of points in the sample space}$. 
The third approach to probability / the third definition is given by:

**Subjective Probability:** This is used when it is difficult to imagine a repetition of an experiment. Example - you get up in the morning and see dark clouds and say - well I am sure that it will rain today - in fact I am 90% certain that it will. In other words, this is an approach based on personal opinion or expertise – another example - statisticians often have to rely on engineers to be able to estimate the type of probability distribution followed by lifetime of products. The problem with this approach is that it varies from person to person and cannot be verified.

Hence we do not use the above approach too often.

Now regardless of what definition/interpretation we use, probability must follow certain rules. These are given as:
Properties of Probabilities

1) From the definition, we can see that for any event, \( A \), if we let \( p = P(A) \), then

\[ 0 \leq p \leq 1. \]

\( p = 0 \) means an event cannot occur, and for \( p = 1 \) means that the event must occur.

So if you roll a die then the probability of rolling a 7 is 0 – WHY????????

On the other hand the probability of rolling a number between 1 and 6 is 1- WHY??????

2) The probabilities of all simple events in a sample space must sum to one.

3) the probability of an event \( A \) is the sum of the probabilities of the individual simple events comprising it.
examples:

1) Toss a coin twice: $S = \{HH, HT, TH, TT\}$

$A = \text{get exactly one head} \ , \ P(A) =$

$B = \text{get at least one head} =$

2) An electronics store has twenty identical CD players out of which one is defective. Suppose you walk in the store and pick one at random. What is the probability that you pick the defective speaker?

Exercises: 3.9, 3.11, 3.15, 3.17, 3.19, 3.20, 3.24
Finally to finish up this section;

Look at example 3.4, pg 119.

- check if it is a valid probability assignment.

A = get exactly one head - calculate $P(A)$

B = get at least one head - calculate $P(B)$
The union of two events, $A$ and $B$, denoted by $A \cup B$ is the event that occurs if an event in at least one of the sets occurs, i.e, $A \cup B$ is the set of all events that are either in $A$ or in $B$ or both.

The intersection of two events, $A$ and $B$ is said to have occurred if both $A$ and $B$ occur on the performance of an experiment. This is denoted by $A \cap B$.

For any event $A$, we define the complement of the event to be the set of all possible outcomes not contained in $A$. The complement is denoted by $A'$ or $A^c$. 
It turns out that, $P(A') = 1 - P(A)$

Two events are defined to be **mutually exclusive** if they have no outcomes in common. In other words, $A \cap B$ is the empty set.
Recall the example with tossing a coin twice.

Here the sample space \( S = \{HH, HT, TH, TT\} \)

Recall \( A = \) get exactly one head =

\( B = \) get at least one head =

Also define \( C = \) get at least one tail =

and \( D = \) get two tails =

Are any of these events mutually exclusive?

Also define in words and in events \( A \cap B, A \cap C, A \cup B, A \cup C, A \cap D, B \cap D \)
**Part of 3.5 General Addition Rule**

For any two events $A$ and $B$,

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

i.e. $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For two mutually exclusive events, we have the **special addition rule**

$$P(A \cup B) = P(A) + P(B)$$
Examples:

1) Let us start with the tossing the two coins example. In this example,

\[ P(A) = \frac{1}{2}, \]

\[ P(B) = \text{________} \]

and \[ P(A \text{ and } B) = \text{________} \]

\[ P(A \cup B) = \text{____________} \]

Also note that A and D are mutually exclusive, therefore,
\[ P(A \cup D) = \text{__________}. \]
2) In a certain suburb of 500 people, 300 are male. Also 50 people are colorblind, out of which 40 are male. If a person is picked at random from the suburb, find the probability of picking either a male or a colorblind person.

\[
P(M \text{ or } CB) = P(M) + P(CB) - P(M \cap CB) = \frac{31}{50}\]
3) In a certain bank, previous records show that out of all prospective customers walking into the bank, 60% open a checking account, 25% open a savings account and 15% open both accounts. Suppose a prospective customer is picked at random. Compute the following probabilities:

a) \( P(\text{customer opens at least one of the two accounts}) = \frac{60}{100} + \frac{25}{100} - \frac{15}{100} = 0.7 \)

b) \( P(\text{customer opens neither of the two accounts}) = 1 - 0.7 = 0.3 \)