

HYPOTHESIS TESTING

CHAPTER 8

Section 8.1 - 8.2

- Large Sample Hypothesis Testing for the mean

Quite often we test hypotheses about statistical parameters. A statistical hypothesis is a claim or a statement about the value of a single parameter or about the values of several parameters.

Examples:

- a) The mean weight of a newborn baby is 9 pounds.
- b) People graduating from FIU get higher paying jobs than those graduating from UM.
- c) Energizer batteries last longer than any other comparable battery.

In any hypothesis testing problem, there are two contradictory hypotheses under consideration. A

statistical test of hypothesis is a method for deciding which of the two hypotheses is correct and is based on the method of proof by contradiction. It is composed of the following five parts:

- 1) The Null Hypothesis, denoted by H_0 .
- 2) The Alternate Hypothesis, denoted by H_a .
- 3) The Test Statistic, denoted by T.S.
- 4) The Rejection Region or the Critical Region, denoted by R.R.
- 5) The Conclusion.

Let us start with an example: Recall the Michelin Tires example. Assume now that you buy the tires from Firestone and that they claim that the mean lifetime of their tires is 41,000 miles. You are not sure and feel that $\mu < 41000$ miles. As before, you test 44 tires and find that $\bar{x} = 40,321$ miles and $s = 2,144$ miles.

1) The hypothesis that you are trying to prove is called the alternate hypothesis. So here you are trying to show that

$H_a :$

(always contains a $<$, $>$, or a \neq sign.)

2) The contradiction of the alternate hypothesis is called the null hypothesis.

Here $H_0 : \mu$

(note that the null hypothesis always contains the $=$ sign)

We prove the alternate by showing that the null is false beyond a reasonable doubt (Think of a murder trial...) . There are only two conclusions we can reach:

Reject the Null

Cannot reject the Null

Summary :

Form of H_0 :
population parameter = hypothesized value
 $\mu = \mu_0$

Form of H_a :

1) $\mu > \mu_0$

2) $\mu < \mu_0$

3) $\mu \neq \mu_0$

Errors in Hypothesis Testing

What are the two most common courtroom mistakes??

a)

In hypothesis testing, this error is the same as rejecting the null hypothesis when it is true. Such an error is called a Type I Error. The probability of making this error is denoted by α and is called the significance level of the test.

b)

In hypothesis testing, this is equivalent to failing to reject the null when it is false. This error is called the Type II error and the probability of making this error is denoted by β .

Tabulated :

	H ₀ True	H ₀ False
Reject H ₀		Correct Decision
Do Not Reject H ₀	Correct Decision	

Ideally, we would like both α and β to be 0. (Recall that these were the probabilities of making the errors.) However, as it turns out, we can control α , but cannot control β . However, decreasing α increases β , and so we have to reach a compromise.

Also, $1 - \beta$ is called the power of the test and it indicates the proportion of all times that the test will actually detect a difference when there is a difference.

Test Statistic: The test statistic is a quantity calculated from the data and works as evidence in favor of the alternate hypothesis. Suppose we test that $H_0 : \mu = \mu_0$, where μ_0 is some hypothesized value of the mean, then if the null is true, we should have $\bar{x} \sim N(\text{_____}, \text{_____})$. Therefore,

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{should have the}$$

_____ distribution.

(Recall that we can substitute σ by s if n is large enough).

For the Michelin Tires example, we have:

$$\bar{x} = \text{_____}, \quad n = \text{_____}, \quad s = \text{_____} \quad \text{and}$$

$$\mu_0 = \text{_____}$$

Thus, the T.S. $Z_0 = \text{_____} = -2.1$

Now the value of the T.S. is extreme or not extreme depending on the alternate and the null hypothesis.

For our alternate hypothesis, we want to see

Again if the null is true, then

$$P(Z \leq -2.1) =$$

Hence, we can safely conclude that:

Rejection Region: This is the region of possible values for which we can reject the null hypothesis. The determination of the region depends on α and the alternate hypothesis. Again, suppose that for our example, we want $\alpha = 0.05$.

Then our rejection region is:

In General:

For n large enough, a fixed α value, and $H_0 : \theta = \theta_0$, we decide the rejection region as follows:

H_a	Support for H_a	Rejection Region
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1) $\mu > \mu_0$

2) $\mu < \mu_0$

3) $\mu \neq \mu_0$

Summary of Hypothesis Testing:

- 1) Describe the population characteristic about which the hypotheses are to be tested.
- 2) State the null and alternate clearly.
- 3) Compute the test statistic.
- 4) Identify the rejection region.
- 5) State the conclusion - it should be stated in the context of the problem and must include the significance level.

Some more examples:

example 2: A supplier supplies components to a manufacturer of electric ovens. The components are supposed to have a heat resistance of 800°F . A sample of 36 such components shows an average heat of resistance of 808°F and a standard deviation of 25°F . Using $\alpha = 0.05$, can we conclude that the heat resistance of the components is greater than 800°F ?

example 3: A coin operated machine is designed to deliver 6 oz. of coffee. The owner believes that the machine is not operating properly. A statistician fills

30 cups and checks for both underfills and overfills. The sample mean is 6.5 oz. and the standard deviation is 0.7 oz. Test with $\alpha = 0.01$.

example 4: A psychologist is interested in knowing whether male heroin addicts' assessment of self worth differs from that of the general male population. On a test designed to measure assessment of self worth, the mean score for the males from the general population is 48.6. A random sample of 30 scores achieved by heroin addicts indicates a score of 44.1 and $s = 6.2$. Do the data indicate that the male heroin addicts value themselves less than the general population? Use $\alpha = 0.01$.