STA 2023
Examples on Probability

3.4 Rules of Probability

General Addition Rule

For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$

i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

For two mutually exclusive events, we have the special addition rule

$P(A \cup B) = P(A) + P(B)$

Hence:

$P(A') = 1 - P(A)$ WHY?

Examples:

1) Let us start with the tossing the two coins example.

Recall that S = {HH, HT, TH, TT}

In this example, Let A = getting exactly one head and B = getting at least one head

and D = getting two tails.

$P(A) =$, $P(B) =$ __________ and $P(A \cap B) =$ ____________

$P(A \cup B) =$ _______________

$P(A \cup D) =$ _____________.

2) In a certain suburb of 500 people, 300 are male. Also 50 people are colorblind, out of which 40 are male. If a person is picked at random from the suburb, find the probability of picking either a male or a colorblind person.

P(M or CB) = P(M) + P(CB) - P($M \cap CB$) = ___________________
= 31/50

3) In a certain bank, previous records show that out of all prospective customers walking into the bank, 60 % open at least a checking account, 25% open at least a savings account and 15% open both accounts. Suppose a prospective customer is picked at random. Compute the following probabilities:

a) P(customer opens at least one of the two accounts) = __________________ = 0.7

b) P(customer opens neither of the two accounts) = ______________

0.3.

Conditional Probability

In certain situation, the sample space S must be changed or altered to take into account new information. In such a situation, probabilities change and are said to be conditional on the occurrence of the event defined by this new information. In some sense, we are basically defining a new sample space S':

So we define the **conditional probability** of the event A given that the event B has occurred. Denoted by P(A|B), it is given by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{provided } P(B) > 0.$$

The formula for computing conditional probabilities leads to a formula for computing the probability of the intersection of two events

The General Multiplication Rule:

$$\begin{aligned} P(\text{ A and B }) &= P(A|B)P(B) \\ &= P(B|A)P(A) \end{aligned}$$

Independence of Events:

Two events A and B are independent if the occurrence of one has no effect on the occurrence of the other. In this situation, **P(A|B) = P(A)** and
P(A and B) = P(A)P(B)

An example The following table shows a breakdown of the drivers in a certain city by sex and by eyesight:

	M	F	Total
Colorblind	500	15	515
Not Colorblind	9235	7800	17,035
Total	9735	7815	17,550

Suppose a driver is chosen at random from the city. Compute the probability that :

a) The person is colorblind:

b) The person is female and colorblind.

c) The person is female given that the person is colorblind.

d) A Suppose that we pick a female. What is the probability that the person is colorblind?

e) Are the events being a female and being colorblind independent?

2) In a certain college 6% of all students are business majors, 50% are female and 10% of the females are business majors. Suppose a student is selected at random. Find the probability that:

1) P(Business|F) =

P(Bus and F) = _______________ = 0.05

P(F|Bus) = ______________ = 0.8333

P(Bus and Male) = _______________

P(M| Bus) = ___________________

3) Suppose we select two machines at random from 5 for inspection. Suppose also that unknown to the inspector, two of the five machines are defective.

a) Find the probability that both machines picked for inspection are defective?

b) Find the probability that at least one of the machines picked for inspection is non defective.

c) Suppose the machines are selected with replacement. How do your answers in (a) and (b) change?

(4) A particular airline flies a 10 AM flight to Chicago from Miami and a 10 AM flight to LA from Miami. Let C be the event that the Chicago flight is full and let L be the event that the LA flight is full. Also $P(C) = 0.6$, $P(L) = 0.5$ and the two events L and C are independent of each other. What is the probability that both flights are full?

Exercises: 3.23, 3.27, 3.29, 3.30, 3.31, 3.33, 3.35, 3.38, 3.39, 3.43, 3.45, 3.46, 3.50, 3.51, 3.53, 3.55, 3.59, 3.63, 3.64