Note: What we have seen here is the Large Sample Inference for \( \mu \). More often than not, the sample size is not large and \( \sigma \) is unknown. In this situation, we have to resort to small sample inference using the t distribution:

Recall that we have said that for large \( n \), the variable \( \frac{\bar{x} - \mu}{s / \sqrt{n}} \) has an approximately normal distribution. However, when \( n \) is not large enough, \( \frac{\bar{x} - \mu}{s / \sqrt{n}} \) does not have a normal distribution even when the original population is normally distributed.
In this situation, it can be shown that if the original population is normal, then \( \frac{\bar{x} - \mu}{s/\sqrt{n}} \) has what is called a **t distribution with n-1 degrees of freedom**, df. The t distribution is symmetric about 0, is bell shaped but has somewhat "fatter" tails than the normal distribution. The exact shape of the t distribution depends on its df. As the df increases, the t distribution approaches the normal distribution.

Look at Figure 7.7 on page 317

As before, define t(n-1), \( \alpha \) to be the point on the t(n-1) curve with area \( \alpha \) above it. These values appear in Table 6, page 796 in the appendix.
**Note:** 1) If the correct df does not appear in the table, usually round down to the closest df.

2) \( t(\infty) = z \). Thus \( Z_\alpha \) values can be obtained from the bottom of this table.

Using a probability statement similar to the one before, we have that a \((1-\alpha)100\%\) confidence interval for \( \mu \) when \( \sigma \) is unknown is:

\[
x \pm t_{(n-1),\alpha/2} \frac{s}{\sqrt{n}}
\]
t-Test:

We still test $H_0: \mu = \mu_0$ against some alternative. This time the test statistic is:

$$t_0 = \frac{(\bar{x} - \mu_0)}{s / \sqrt{n}}$$

For $H_a: \mu > \mu_0$, Reject $H_0$ if $t_0 > t_{\alpha}, n-1$

For $H_a: \mu < \mu_0$, Reject $H_0$ if $t_0 < -t_{\alpha}, n-1$

For $H_a: \mu \neq \mu_0$, Reject $H_0$ if $|t_0| > t_{\alpha/2}, n-1$
Example:

A study conducted in England found that sleep deprivation sabotages the ability of adults to perform simple everyday tasks. 12 adults were randomly picked and deprived of one night’s sleep. They then had to take a test to measure their ability to perform simple tasks. Suppose that the scores of the 12 adults in this study were as follows: \( \bar{X} = 63 \) and sample standard deviation, \( s = 17 \).

Do we have evidence to believe that the average score of sleep deprived adults on this test is lower than 80, the score of adults who have slept the previous night? Use \( \alpha = 0.05 \). Also find a 95% confidence interval for the true mean score of sleep deprived adults on this test.
Exercises: 7.25, 7.26, 7.31, 7.33, 7.37, 7.42, 8.59, 8.61, 8.63; 8.64, 8.65