Conditional Probabilities

• Looking at our 5 management types, let’s ask a harder question.
• What proportion of assistant managers are male?
• Another way to ask this question is, what is the probability that a male is chosen given that an assistant manager has been chosen?
• This probability is denoted as $P(A|B)$, which is read as the probability of A given that B has occurred.
• This type of probability is called a conditional probability.
Since we know that B has occurred, we act as if Kim and David are no longer in our set. There are 3 people left and two of them are males, so the $P(A|B) = \frac{2}{3}$. 
Since we know that A has occurred, we act as if Kim and Maria are no longer in our set. There are 3 people left and two of them are assistant managers, so the P(A|B) = 2/3.
Since we know that B has occurred, we act as if Kim and David are no longer in our set. There are 3 people left and one of them is not male, so the $P(A^c|B) = 1/3$. 

$$P(A^c | B)$$
How are Conditional Probabilities Interpreted?

- $P(A|B) = \frac{2}{3}$. We say that 2 thirds of the assistant managers are male.

- $P(B|A) = \frac{2}{3}$. We say that 2 thirds of the males are assistant managers.

- $P(A^c|B) = \frac{1}{3}$. We say that 1 third of the assistant managers are not male.
Independent Events

- Two events A and B are independent if knowing that one has occurred does not change the probability that the other occurs.
- We need a mathematical expression of this relationship.
- Two events A and B that both have positive probability are independent if \( P(A) = P(A|B) \) (or \( P(B) = P(B|A) \)).
What is the probability that a randomly selected male who never smoked cigars died of a smoking related cancer?

Let’s pose the question another way.

Given that the man never smoked cigars, what is the probability that he died of a smoking related cancer?

Let’s pose the question yet another way.

What proportion of the men who never smoked cigars died of smoking related cancers?
What is the probability that a randomly selected male who never smoked cigars died of a smoking related cancer (P(D|A))?

<table>
<thead>
<tr>
<th>Cigars</th>
<th>Died from Cancer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes (D)</td>
</tr>
<tr>
<td>Never Smoked (A)</td>
<td>782</td>
</tr>
<tr>
<td>Former Smoker (B)</td>
<td>91</td>
</tr>
<tr>
<td>Current Smoker (C)</td>
<td>141</td>
</tr>
<tr>
<td>Totals</td>
<td>1,014</td>
</tr>
</tbody>
</table>

\[
P(D | A) = \frac{P(A \cap D)}{P(A)} = \frac{\frac{782}{137,243}}{\frac{121,529}{137,243}} = \frac{782}{121,529}
\]
Are the events A and D independent?

To answer this question we must compare two probabilities, either \( P(D) \) and \( P(D|A) \) or \( P(A) \) and \( P(A|D) \).

Since we have already computed \( P(D|A) \), I will compare \( P(D) \) and \( P(D|A) \).

If these two probabilities are equal, then A and D are independent events.
Compare $P(D)$ and $P(D|A)$.

$$P(D \mid A) = \frac{P(A \cap D)}{P(A)} = \frac{782}{137,243} = \frac{782}{121,529}$$

$$P(D \mid A) = 0.006434678 \ldots$$

$$P(D) = \frac{1,014}{137,243} = 0.007388354 \ldots$$

Since $P(D) \neq P(D|A)$, the events $A$ and $D$ are not independent.