Conditional Probabilities

- Looking at our 5 management types, let’s ask a harder question.
- What proportion of assistant managers are male?
- Another way to ask this question is, what is the probability that a male is chosen given that an assistant manager has been chosen?
- This probability is denoted as $P(A|B)$, which is read as the probability of $A$ given that $B$ has occurred.
- This type of probability is called a conditional probability.

P(A|B)

Since we know that $B$ has occurred, we act as if Kim and David are no longer in our set. There are 3 people left and two of them are males, so the $P(A|B) = \frac{2}{3}$.

P(B|A)

Since we know that $A$ has occurred, we act as if Kim and Maria are no longer in our set. There are 3 people left and two of them are assistant managers, so the $P(A|B) = \frac{2}{3}$.

P(A^c|B)

Since we know that $B$ has occurred, we act as if Kim and David are no longer in our set. There are 3 people left and one of them is not male, so the $P(A^c|B) = \frac{1}{3}$.

How are Conditional Probabilities Interpreted?

- $P(A|B) = \frac{2}{3}$. We say that 2 thirds of the assistant managers are male.
- $P(B|A) = \frac{2}{3}$. We say that 2 thirds of the males are assistant managers.
- $P(A^c|B) = \frac{1}{3}$. We say that 1 third of the assistant managers are not male.

Independent Events

- Two events $A$ and $B$ are independent if knowing that one has occurred does not change the probability that the other occurs.
- We need a mathematical expression of this relationship.
- Two events $A$ and $B$ that both have positive probability are independent if $P(A) = P(A|B)$ (or $P(B) = P(B|A)$).
What is the probability that a randomly selected male who never smoked cigars died of a smoking related cancer?

Let's pose the question another way.

Given that the man never smoked cigars, what is the probability that he died of a smoking related cancer?

Let's pose the question yet another way.

What proportion of the men who never smoked cigars died of smoking related cancers?

\[
P(D \mid A) = \frac{P(A \cap D)}{P(A)} = \frac{\frac{782}{137,243}}{\frac{121,529}{137,243}} = \frac{782}{121,529} \\
\]

Are the events A and D independent?

To answer this question we must compare two probabilities, either \( P(D) \) and \( P(D \mid A) \) or \( P(A) \) and \( P(A \mid D) \).

Since we have already computed \( P(D \mid A) \), I will compare \( P(D) \) and \( P(D \mid A) \).

If these two probabilities are equal, then A and D are independent events.

\[
P(D \mid A) = \frac{P(A \cap D)}{P(A)} = \frac{\frac{782}{137,243}}{\frac{121,529}{137,243}} = \frac{782}{121,529} \\
\]

\[
P(D) = \frac{1,014}{137,243} = .007388354 ... \\
\]

\[
P(D) \neq P(D \mid A), \text{ the events A and D are not independent.} \\
\]