Randomized Block Design

What are blocks?

• A block is a set of matched experimental units.

• Blocks are formed by matching experimental units as closely as possible based on factors that are known to affect the response variable.
Steps in a RBD

• First form blocks of matched experimental units.
• If there are p levels of the treatment, then each block should consist of p experimental units.
• After forming the blocks, one experimental unit from each block is randomly assigned to each treatment level.
\[ SS \text{ (TOTAL)} = SS_{xx} = \sum_{i=1}^{p} \sum_{j=1}^{b} (x_{ij} - \bar{x})^2 \]

\[ SS \text{ (TOTAL)} = SST + SSB + SSE \]

\[ \bar{x} = \frac{\sum_{i=1}^{p} \sum_{j=1}^{b} x_{ij}}{n}, \text{ where } n = pb \]

\[ SST = \sum_{i=1}^{p} (\bar{x}_{T_i} - \bar{x})^2 \]

\[ SSB = \sum_{j=1}^{b} (\bar{x}_{B_j} - \bar{x})^2 \]

\[ SSE = SS \text{ (TOTAL)} - SST - SSB \]
The ANOVA SUMMARY TABLE: Randomized Block Design

\[ p = \# \text{ of treatments and } b = \# \text{ of blocks. } n = bp \]

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>p-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blocks</td>
<td>b-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>(p-1)(b-1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>n-1</td>
<td>bp - 1</td>
<td>bp</td>
<td></td>
</tr>
</tbody>
</table>
What is the advantage of blocking?

If there is a known source of variability, carefully matched blocks will permit us to reduce the variability due to error.
How does blocking reduce error?

- \( SS(\text{Total}) = SST + SSB + SSE \)

- If a completely randomized design is used, \( SS(\text{Total}) = SST + SSE \)

- If the observed values in each treatment level for the completely randomized design (CRD) and the randomized block design (RBD) are the same, then \( SSE(\text{CRD}) = SSB + SSE(\text{RBD}) \)
Some Notes

• The are two possible F-tests in a RBD.
• The F for treatments is used to determine whether the means differ for at least 2 of the p treatments.
• The F for blocks is used to determine whether blocking was effective.
• The F test for blocks is not usually done because we block on a factor which we know affects the response.