Formulas for One-way ANOVA and Power

Completely Randomized Design

\[ N = n_1 + n_2 + \ldots + n_k \quad \text{and} \quad \bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \]

\[ SS_{\text{total}} = SS_{\text{xx}} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 = \sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 - \left( \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}}{N} \right)^2 \]

\[ SS_{\text{treat}} = \sum_{j=1}^{k} n_j (\bar{x}_j - \bar{x})^2 \]

\[ SS_{\text{error}} = \sum_{j=1}^{k} (n_j - 1) s_j^2 = SS_{\text{total}} - SS_{\text{treat}} \]

\[ MS_{\text{treat}} = \frac{SS_{\text{treat}}}{k-1} \quad \text{and} \quad MS_{\text{error}} = \frac{SS_{\text{error}}}{N-k} \]

\[ F = \frac{MS_{\text{treat}}}{MS_{\text{error}}} , \quad df_{\text{treat}} = k-1 \quad df_{\text{t}} = N-k \]

\[ \eta^2 = \frac{SS_{\text{treat}}}{SS_{\text{total}}} \quad \omega^2 = \frac{\frac{\sigma_t^2}{\sigma_t^2 + \sigma_e^2}} \]

Random effects model: \[ \hat{\omega}^2 = \frac{MS_{\text{treat}} - MS_{\text{error}}}{MS_{\text{treat}} + (n-1) MS_{\text{error}}} \]

Fixed effects model: \[ \phi^2 = \frac{SS_{\text{treat}} - (k-1) MS_{\text{error}}}{SS_{\text{total}} + MS_{\text{error}}} \]

\[ \phi = \sqrt{\frac{\sum_{j=1}^{k} n_j \tau_j^2}{k\sigma_e^2}} \]

For \( n_1 = n_2 = \ldots = n_k = n \), \( \phi = \phi' \sqrt{n} \) where \( \phi' = \sqrt{\frac{\sum_{j=1}^{k} \tau_j^2}{k\sigma_e^2}} \)

Let \( \Delta = \mu_{\text{max}} - \mu_{\text{min}} \), then \( \phi' = \frac{\Delta^2}{2k\sigma_e^2} \)