Experiments

Experiment
- 1. Investigator Controls One or More Independent Variables
  - Called Treatment Variables or Factors
  - Contain Two or More Levels (Subcategories)
- 2. Observes Effect on Dependent Variable
  - Response to Levels of Independent Variable
- 3. Experimental Design: Plan Used to Test Hypotheses

Examples of Experiments
- 1. Thirty Stores Are Randomly Assigned 1 of 4 Store Displays to See the Effect on Sales.
- 2. Two Hundred Consumers Are Randomly Assigned 1 of 3 Brands of Juice to Study Reaction.

Experimental Designs

- Completely Randomized
- Randomized Block
- Factorial
- One-Way ANOVA
- Randomized Block F Test
- Two-Way ANOVA

Completely Randomized Design
Completely Randomized Design

1. Experimental Units (Subjects) Are Assigned Randomly to Treatments
   - Subjects are Assumed Homogeneous
2. One Factor or Independent Variable
   - 2 or More Treatment Levels or Classifications
3. Analyzed by One-Way ANOVA

Randomized Design Example

<table>
<thead>
<tr>
<th>Factor (Training Method)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent variable (Response)</td>
<td>21 hrs.</td>
<td>17 hrs.</td>
<td>31 hrs.</td>
</tr>
<tr>
<td></td>
<td>27 hrs.</td>
<td>25 hrs.</td>
<td>28 hrs.</td>
</tr>
<tr>
<td></td>
<td>29 hrs.</td>
<td>20 hrs.</td>
<td>22 hrs.</td>
</tr>
</tbody>
</table>

Experimental Designs

One-Way ANOVA F-Test

1. Tests the Equality of 2 or More (p) Population Means
2. Variables
   - One Independent Variable
     - 2 or More (p) Treatment Levels or Classifications
     - One Interval Dependent Variable
3. Used to Analyze Completely Randomized Experimental Designs

One-Way ANOVA F-Test Assumptions

1. Randomness & Independence of Errors
   - Independent Random Samples are Drawn
2. Normality
   - Populations are Normally Distributed
3. Homogeneity of Variance
   - Populations have Equal Variances
One-Way ANOVA F-Test Hypotheses

- $H_0$: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_p$
- All Population Means are Equal
- No Treatment Effect
- $H_a$: Not All $\mu_j$ Are Equal
- At Least 1 Pop. Mean is Different
- Treatment Effect
- $\mu_i \neq \mu_j$ for some $i \neq j$
- $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_p$ is Wrong

One-Way ANOVA

Basic Idea

1. Compares 2 Types of Variation to Test Equality of Means
2. Comparison Basis Is Ratio of Variances
3. If Treatment Variation Is Significantly Greater Than Random Variation then Means Are Not Equal
4. Variation Measures Are Obtained by ‘Partitioning’ Total Variation

Total Variation

$SS(\text{total}) = (x_{11} - \bar{X})^2 + (x_{21} - \bar{X})^2 + \ldots + (x_{ij} - \bar{X})^2$

Response, $X$

Group 1 Group 2 Group 3

Treatment Variation

$SST = n_1(\bar{X}_1 - \bar{X})^2 + n_2(\bar{X}_2 - \bar{X})^2 + \ldots + n_k(\bar{X}_k - \bar{X})^2$

Response, $X$

Group 1 Group 2 Group 3
Random (Error) Variation

\[ SSE = (X_{11} - \bar{X}_1)^2 + (X_{21} - \bar{X}_2)^2 + \ldots + (X_{p1} - \bar{X}_p)^2 \]

Response, X

Group 1  Group 2  Group 3

One-Way ANOVA F-Test

Test Statistic

1. Test Statistic
   - \( F = \frac{MST}{MSE} \)
     - MST is Mean Square for Treatment
     - MSE is Mean Square for Error

2. Degrees of Freedom
   - \( \nu_1 = p - 1 \)
   - \( \nu_2 = n - p \)
   - \( p = \) # Populations, Groups, or Levels
   - \( n = \) Total Sample Size

One-Way ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square (Variance)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>p - 1</td>
<td>SST</td>
<td>MST = SST/(p - 1)</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>n - p</td>
<td>SSE</td>
<td>MSE = SSE/(n - p)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>n - 1</td>
<td>SS(Total) = SST+SSE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

One-Way ANOVA F-Test Critical Value

If means are equal,
\[ F = \frac{MST}{MSE} \approx 1. \]
Only reject large \( F \)

One-Way ANOVA F-Test Example

As production manager, you want to see if 3 filling machines have different mean filling times. You assign 15 similarly trained & experienced workers, 5 per machine, to the machines. At the .05 level, is there a difference in mean filling times?

Mach1  Mach2  Mach3
25.40  23.40  20.00
26.31  21.80  22.20
24.10  23.50  19.75
23.74  22.75  20.60
25.10  21.60  20.40

One-Way ANOVA F-Test Solution

- \( H_0: \mu_i = \mu_j \) for some \( i \neq j \)
- \( H_a: \mu_1 \neq \mu_2 \neq \mu_3 \)
- Test Statistic: \[ F = \frac{MST}{MSE}, \nu_1 = 2, \nu_2 = 12 \]
- \( \alpha = .05 \)
- \( RR: F > F_{.05, 2, 12} = 3.89 \)
One-Way ANOVA F-Test Solution

Calculations:
\[ F = \frac{MST}{MSE} = \frac{23.5820}{.9211} = 25.6 \]

Decision: Reject Ho.
Conclusion: There is enough evidence to indicate that the mean filling times differ for at least 2 of the 3 machines.

Summary Table Solution

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square (Variance)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (Machines)</td>
<td>3 - 1 = 2</td>
<td>47.1640</td>
<td>23.5820</td>
<td>25.6</td>
</tr>
<tr>
<td>Error</td>
<td>15 - 3 = 12</td>
<td>11.0532</td>
<td>.9211</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15 - 1 = 14</td>
<td>58.2172</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Computer