1) \( H_a: \) The lifetime of a cutting tool is linearly related to the speed at which it is operated. \((\beta_1 \neq 0)\)

\[ H_o: \] The lifetime of a cutting tool is not linearly related to the speed at which it is operated. \((\beta_1 = 0)\)

Assumptions:
- The cutting tools were randomly and independently chosen.
- \(\varepsilon\) is normally distributed. \(\mu_\varepsilon = 0\) and \(\sigma_\varepsilon^2 = \sigma^2\).
- Note: \(\sigma^2\) is not a function of the operating speed.

Test Statistic: \( t = \frac{b_1}{s_{b_1}},\) \( df = n-1\) \( df = 28\)

\(\alpha = .05\) \( RR: t < -2.048\) or \( t > 2.048\) (p-value of \( t \leq .05\))

Calculations: From SPSS, \( t = -6.497\) with p-value = .0000

Decision: Reject \( H_o.\)

Conclusion: The data provide enough evidence to indicate that the lifetime of a cutting tool is linearly related to the speed at which it is operated.

2) \( \hat{y} = 7.96500 - .09017x, \) for \(30 < x < 70\)

3) \( r^2 = .60123.\) 60.123\% of the variability in the lifetime of these cutting tools is explained by the estimated linear relationship between the life of the tool and the speed at which it is operated.

5) Lack of Linear Fit Test

\[ F = \frac{SSE(reg) - SSE(ANOVA)}{(n-2) - (n-p)} \]

Test Statistic: \( df_\text{numerator} = p-2\) \( df_\text{denominator} = n-p\)

\( n-2 = 28, n-p = 25, p-2 = 3\)

\(\alpha = .05\) \( RR: F \geq F_{.05, 3, 28} = 2.95\)

Calculations: \( F = \frac{32.35350 - 31.758}{(28) - (25)} = 1.270\) \( = .156\)
Decision: Do not reject $H_0$.

Conclusion: Given that the linear model fits the data, there is not enough evidence to indicate that the relationship is nonlinear in operating speed.

6) $95\%$ CI for $\beta_1$
   
   
   $b_1 \pm t_{\alpha/2} s_{b_1}$
   
   $.090167 \pm 2.048(.013877)$
   
   $.090167 \pm .028420$
   
   $-.118587 < \beta_1 < -.061747$

From SPSS output

$-.118593 < \beta_1 < -.061740$

We are $95\%$ sure that if the speed at which the cutting tool is operated is increased by 1 meter per minute, the lifetime of the cutting tool will decrease by between .06 and .12 hours, on the average.

7) $2.06264 < \mu_{y|x=60} < 3.04736$

We are $95\%$ sure that the mean lifetime for all cutting tools operated at 60 meters per minute will be between 2.06 and 3.05 hours.

8) $1.66469 < Y_{x=45} < 6.15031$

We are $95\%$ sure that the next cutting tool operated at 45 meters per minute will last between 1.66 and 6.15 hours.