Linear Contrasts

\[ \psi_i = a_{i1} \mu_1 + a_{i2} \mu_2 + \ldots + a_{ip} \mu_p, \text{ where } \sum a_{ij} = 0 \]

\[ L_i = a_{i1} \bar{x}_1 + a_{i2} \bar{x}_2 + \ldots + a_{ip} \bar{x}_p \]

\[
\text{var}(L_i) = \sum_{j=1}^{p} a_{ij}^2 \text{var}(\bar{x}_j) = \sigma^2 \sum_{j=1}^{k} \frac{a_{ij}^2}{n_j}
\]

\[
\text{est. var}(L_i) = MS_{\text{error}} \sum_{j=1}^{k} \frac{a_{ij}^2}{n_j}
\]

Test Statistic: \[ t = \frac{L_i - \psi_i}{\sqrt{MS_{\text{error}} \sum_{j=1}^{k} \frac{a_{ij}^2}{n_j}}}, \quad df = \nu = N - p \]

Confidence Interval for \( \psi_i \): \[ L_i \pm t_{\alpha/2} \sqrt{SS(L_i)} \]

\[
SS(L_i) = \frac{L_i^2}{\sum_{j=1}^{k} \frac{a_{ij}^2}{n_j}}
\]

\[
F_{\nu_1, \nu_2} = \frac{SS(L_i)}{MS_{\text{error}}}, \quad \nu_1 = 1, \quad \nu_2 = N - p
\]
Inferences for Linear Contrasts (Comparisons)

\[ H_0: \ \psi_g = 0 \quad H_a: \ \psi_g \neq 0 \]

Test Statistic: \[ t = \frac{L_g - 0}{\sqrt{\frac{MSE}{\sum_{j=1}^{k} \frac{a_j^2}{n_j}}}} \]

or \[ F = \frac{L_g^2}{MSE \sum_{j=1}^{k} \frac{a_j^2}{n_j}} \]

RR: \[ |t| > t_{\alpha_{rc}} \quad \nu = N - k \quad \text{or} \quad F > F_{\alpha_{rc}} \quad \nu_1 = 1 \quad \text{and} \quad \nu_2 = N - k \]

\[ H_0: \ \psi_g \geq 0 \quad H_a: \ \psi_g < 0 \quad H_0: \ \psi_g < 0 \quad H_a: \ \psi_g > 0 \]

RR: \[ t < -t_{\alpha_{rc}} \quad \text{RR: } t > t_{\alpha_{rc}} \]

Confidence Interval for \( \Psi_g \):

\[ L_g - t_{\alpha_{rc}, \frac{\nu}{2}} \sqrt{MSE \sum_{j=1}^{k} \frac{a_j^2}{n_j}} < \psi_g < L_g + t_{\alpha_{rc}, \frac{\nu}{2}} \sqrt{MSE \sum_{j=1}^{k} \frac{a_j^2}{n_j}} \]

Let \( c \) = \# of comparisons.

Orthogonal Contrasts: \[ \alpha_{PC} = 1 - (1 - \alpha_{SW})^\frac{1}{c} \]

Nonorthogonal Contrasts: \[ \alpha_{PC} = \frac{\alpha_{SW}}{c} \]

Post-Hoc Contrasts: Use the Scheffé method:

Replace \( t_{\alpha_{rc}, \frac{\nu}{2}} \) by \( S \) and \( F_{\alpha_{rc}} \) by \( S^2 \), where \( S = \sqrt{(k - 1) F_{\alpha, k-1, N-k}} \).
**Pairwise Comparisons (All Possible Pairs)**

H₀: Ψ = μᵢ - μⱼ = 0, for all i ≠ j  
Hₐ: Ψ = μᵢ - μⱼ ≠ 0, for i ≠ j

Test Statistic: \( D = |L| = |\bar{x}_i - \bar{x}_j| \)

RR: \( D > CV \) (the critical value)

Confidence Interval: \( L ± CV = \bar{x}_i - \bar{x}_j ± CV \)

**Bonferroni:**  
\[
CV = t_{\alpha \frac{k(k-1)}{2}} \sqrt{MSE \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}, \quad \text{where} \quad c = k(k-1)/2
\]

**Scheffé:**  
\[
CV = S \sqrt{MSE \left( \frac{1}{n_i} + \frac{1}{n_j} \right)} = \sqrt{(k-1) \frac{F_{\alpha, k-1, n-k}}{MSE} \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}
\]

**Tukey's HSD:**  
\[
CV = q_{\alpha, k, n-k} \sqrt{MSE \frac{1}{\bar{F}}}
\]

**SNK:**  
\[
CV = q_{\alpha, r, n-k} \sqrt{MSE \frac{1}{\bar{F}}} \quad r = 2, 3, \ldots \text{ or } p
\]

If the samples sizes are equal, then \( \tilde{n} = n_1 = n_2 = \ldots = n_k \). If the sample sizes are not equal, there are three different formulas for \( \tilde{n} \):

1) \( \tilde{n} = \frac{k}{\frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_k}} = \frac{k}{\sum_{j=1}^{k} \frac{1}{n_j}} \)

2) \( \tilde{n} = \frac{2n_i n_j}{n_i + n_j} \)

3) \( \tilde{n} = \frac{2n_m n_s}{n_m + n_s} \)

where \( n_m \) is the size of the sample with the largest mean and \( n_s \) is the size of the sample with the smallest mean. The formula used depends upon the software package.