Hypothesis Test Example for Binomial parameter $p$

Exercise 8.60 page 319 in *Statistics, 7th Edition* by McClave, Dietrich and Sincich: "Earthquakes are not uncommon in California. An article in the *Annals of the Association of American Geographers* (June 1992) investigated many factors that California residents consider when purchasing earthquake insurance. The survey revealed that only 133 of 337 randomly selected residences in Los Angeles County were protected by earthquake insurance."

Do the data provide sufficient evidence to indicate that less than 40% of the residents of Los Angeles County were protected by earthquake insurance? Use $\alpha = .10$.

**H$_a$:** Less than 40% of the residents of Los Angeles County are protected by earthquake insurance. ($p < .4$)

**H$_0$:** At least 40% of the residents of Los Angeles County are protected by earthquake insurance. ($p \geq .4$)

**Assumptions:** A binomial experiment with $n = 337$ was done. Is $n = 337$ large enough to indicate that the sampling distribution of $\hat{p}$ is approximately normal? Does the interval $p_o \pm 3\sqrt{\frac{p_o q_o}{n}}$ contain either 0 or 1?

Since $p_o = .4$, $p_o \pm 3\sqrt{\frac{p_o q_o}{n}} = .4 \pm 3\sqrt{\frac{.4(.6)}{337}} = .4 \pm .080 = (.320, .480)$. Neither 0 nor 1 is in this interval. Hence, the sampling distribution of $\hat{p}$ is approximately normal.

**Test Statistic:**

$$z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$$

$\alpha = .10$  
**RR:** $z < -1.28$

**Calculation:** $x = 133$ and $n = 337$

$$\hat{p} = \frac{x}{n} = \frac{133}{337}, \quad z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}} = \frac{\frac{133}{337} - .4}{\sqrt{\frac{.4(.6)}{337}}} = -.200.$$  

**Decision:** Do not reject $H_0$.

**Conclusion:** There is not sufficient evidence to indicate that less than 40% of the residents of Los Angeles County are protected by earthquake insurance.

The **p-value of the test statistic** is $P(z < -.20) = .5 - .0793 = .4207$. If the null hypothesis is true, there is a 42.07% chance of finding a value of $z$ which is less than or equal to -.20. Hence, there is not sufficient evidence to indicate that less than 40% of the residents of Los Angeles County are protected by earthquake insurance.