

1) [25pt] Short answer problems:

- a) Solve for x , given that $\log_2 x^3 = 6$.
- b) Given that $\sec \theta = -5/2$ and that $\pi/2 < \theta < \pi$, compute $\tan \theta$.
- c) Solve $x^2 - 4x + 3 < 0$.
- d) State the domain and range of $f(x) = 2 + \sqrt{x - 3}$.
- e) Define instantaneous velocity (I am expecting a short formula with a short explanation).

2) [10pt] Sketch the curve by eliminating the parameter and show the direction of movement; $x = \sqrt{t}$, $y = 2t + 4$.

3) (15pts) Answer True or False. You do not have to explain.

- a) $\tan(\pi + x)$ is continuous at every point in the interval $(0, \pi)$.
- b) If $\lim_{x \rightarrow 2} f(x)$ exists, then $\lim_{x \rightarrow 2^+} f(x)$ exists.
- c) If x is a number so that $|x - 2| < 1$, then $|x - 4| < 5$.
- d) $\forall \epsilon > 0, \exists \delta > 0, \delta + \epsilon < .001$.
- e) The Earth's population is a continuous function of time.

4) (25pt) Compute the following limits;

- a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} =$
- b) $\lim_{t \rightarrow 2^-} \frac{|t - 2|}{t - 2} =$
- c) $\lim_{x \rightarrow +\infty} \frac{5x^2 + 25x + 5}{3x^2 - 75} =$
- d) $\lim_{x \rightarrow 3^+} \pi^2 =$
- e) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} =$

5) [10pt] Answer these using the graph below (it's based on the first figure on page 121).

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{3} =$$

$$\lim_{x \rightarrow 2^+} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$f(2) =$$

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} =$$

6) [15pts] Choose ONE of the problems below to do. Remember to use enough words and sentences - not just formulas. You can continue on the back.

a) Show that $\sin(x)$ is continuous [you may assume, as in class, that \sin and \cos are continuous at 0].

b) Show that $\lim_{x \rightarrow 3} 2x - 1 = 5$ using the definition of limit.

c) [5pts bonus, if correct] Show that $\lim_{x \rightarrow 3} x^2 = 9$ using the definition of limit.

Answers and Remarks:

1a) $x = 4$

1b) $-\sqrt{21}/2$

1c) $x \in (1, 3)$

1d) $D = [3, +\infty)$ and $R = [2, +\infty)$

1e) The formula is $i.\text{vel} = \lim \text{avg. vel}$ (and include an explanation).

2) $y = 2x^2 + 4$. The graph is the right half of a parabola (with an arrow pointing "north-east").

3) FTTF. Someone asked about 3e): Since the population is always an integer, it cannot change gradually. However, when the numbers involved are so large, we might sometimes pretend that the function is continuous [to be more precise - we might approximate the population with a formula such as $\exp(20 + t/100)$ which is continuous].

If you didn't do well on these, practice TF before next exams [see my exam page, the book + supplements, etc]. This will help you get used to the precise language of advanced mathematics (and probably help your grade, too).

4) 6, -1, $5/3$, π^2 , $1/4$

5) $-2/3$, 0, 0, 1, 1

We went over this in class Monday. It is a simple version of Ch 2.2 problem 2 (try that one!). For some reason, the grades were low, so I may ask a question like this on Exam 2.

6) See the book + lectures for these. In option-problems like this one, just choose ONE part, so I can grade you fairly. For THIS exam only, I gave +1 to people who did c) correctly, after doing a) or b).

Remark on a): The proof is centered around a calculation, but you should try to explain almost every step. For example, one important step is $\lim_{h \rightarrow 0} \sin(h) = 0$. At that point write something like "because $\sin(x)$ is continuous at 0".

Remark on b) and c): I am pretty happy with the formulas I saw. Remember that most proofs are a sequence of *sentences* (which may include formulas) and the words you choose are very important to whoever reads the proof. For example, there is a huge difference in meaning between these:

"Let $\epsilon > 0$."

"If $\epsilon > 0 \dots$ "

"Since $\epsilon > 0 \dots$ " and

"So, $\epsilon > 0$."