MAC 2311 Feb 7, 2005
Exam I and Key Prof. S. Hudson

1) [10 pts] Show your work, and explain briefly.

a) Find the solution set of: $1/x < 1/(x+1)$.

b) Find the slope-intercept form of the line perpendicular to $y = 5x + 9$ that has y-intercept 6.

2) [15pts] Find a value for c that makes f continuous and then graph f.

$$f(x) = \begin{cases} 2x + c & \text{if } x < 1 \\ 5 & \text{if } x \geq 1 \end{cases}$$

3) [10 pts] A stone dropped into a pond at time $t = 0$ causes a circular ripple that travels out from the center at 4 feet per second. At what rate (in square feet per second) is the area within the circle increasing when $t = 5$?

4) [10 pts] Show your work, and explain briefly.

a) Use $\pi/3 + \pi/4 = 7\pi/12$ and a trig identity to compute $\cos(7\pi/12) =$. Do all the trig necessary to simplify this, but you can leave fractions and square root signs in your answer.

b) Find the largest possible value for δ that makes the following statement true: If $|x - 3| < \delta$ then $|(2x + 1) - 7| < 0.0004$.

5) [20 pts] Compute the following limits;

a) $\lim_{x\to 2} \frac{x-2}{x^2+x-6} =$

b) $\lim_{x\to+\infty} \frac{3x+2}{5x^2-2} =$

c) $\lim_{t\to 0^+} \frac{|t|}{t} =$

d) $\lim_{x\to 3^+} \pi^2 =$

6) [15 pts] Choose ONE of the proofs below to do.

a) Prove that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.

b) Use the I.V.Thm to prove that $x^3 - 4x + 3 = 0$ has *more than one* root.

c) Use the definition of derivative to prove that $D_x(ax^2 + bx + c) = 2ax + b$.

7) [20 pts] Answer True or False. You do not have to explain. The first two refer to a ball tossed upwards and caught on the way down, and how time, height and velocity are

related to each other.

a) Velocity is a function of height.

b) Time is a function of height.

c) Average velocity is the slope of a tangent line.

d) $\forall \epsilon > 0, \exists \delta > 0$ such that $3\delta < \epsilon$.

e) If $\lim_{x \to 2^+} f(x)$ exists, then $\lim_{x \to 2} f(x)$ exists.

BONUS [5 pt]: Prove that $\lim_{x \to 4} x^2 = 16$ using the ϵ method. If you answer on the back, leave a brief note here on this page.

Answers

After removing some very low scores, the average was about 60/100. So, the scale can be lowered about 5 points. The lowest scores occurred on problems 1a and 3.

1a) There are several possible methods, but I recommend looking for endpoints first, and drawing them on a number line (picture omitted). It is impossible to solve $1/x = 1(x+1)$, so the only endpoints are the two discontinuities, $x = 0$ and $x = -1$. A little testing shows the solution set is the interval $(-1, 0)$ [done in class].

1b) $y = -x/5 + 6$

2) Clearly, f will be continuous *except* maybe at $x = 1$. Set $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$. Then $2 \cdot 1 + c = 5$ and $c = 3$. [9 pts. Mentioning "lim" was 2 pts. The graph was worth 6pts].

3) We are told that $dr/dt = 4$ and are asked about dA/dt. Avoid the notation A' because it might mean dA/dr (a common mistake was to mix up dA/dt with dA/dr). You should know that $A = \pi r^2$, but you have to figure out the simple relation between r and t (to plug in for r).

Since $r(t)$ is linear, $r = mt + b$. Since $b = r(0) = 0$ and $m = r'(t) = 4$, we get $r(t) = 4t$. So, $A = \pi r^2 = 16\pi t^2$, and $dA/dt = 32t$. Set $t = 5$ and get 160π ft^2 /sec.

It is also possible to solve this with the Chain Rule.

4a) $\cos(7\pi/12) = (\sqrt{2} - \sqrt{6})/4$.

4b) $\delta = 0.0002$

5) 1/5, 0, 1, π^2

6a) and 6c) are in the book. Remember to explain your proofs with words.

6b) We compute $f(1) = 0$ (so, one root is $x = 1$). Also, $f(-1) = -2 < 0$ and $f(0) = 3 > 0$ and f is continuous. So, by the IVT, there is a second root between -1 and 0.

7) FFFTF

Bonus: I did one like this in class including the details. Some key steps only: If $|x - 4| < 1$ then (algebra omitted) $|x + 4| < 9$. We can set $\delta = \min\{1, \epsilon/9\}$ and base a proof on that.