

1) [25pt] Short answer problems:

a) Solve for  $x$ , given that  $|x - 1| < x + 1$ .

b) Solve for  $x$ , given that  $\cos(2x) = 0$ .

c) Solve for  $x$ , given that  $\log_{10}(1 + x) = 3$ .

d) Complete the identity (no explanation required):  $\sin(2\alpha) =$

e) Complete the identity (calculate, using a triangle):  $\sin(\cos^{-1} x) =$

2) [10pt] Find parametric equations for the portion of the parabola  $x = y^2$  joining  $(1,-1)$  to  $(1,1)$ , oriented upwards.

3) [15pts] Answer True or False. You do not have to explain.

a) If  $f$  is continuous at 2, then  $\lim_{x \rightarrow 2^+} f(x)$  exists.

b) If  $\lim_{x \rightarrow 2^-} f(x)$  exists, then  $\lim_{x \rightarrow 2^+} f(x)$  exists.

c) If  $x$  is a number so that  $|x - 2| < 1$ , then  $|x - 4| < 5$ .

d)  $\forall \epsilon > 0, \exists \delta > 0, \delta + \epsilon < .001$ .

e) A rational function is continuous at every point where the denominator is nonzero.

4) (25pt) Compute the following limits. Notice that 3 are one-sided. You may have to answer, for example, with 'd.n.e.' or '+ $\infty$ ' [the latter will get more credit, if it is correct].

a)  $\lim_{x \rightarrow 3^+} \frac{x}{x^2 - 9} =$

b)  $\lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1} =$

c)  $\lim_{x \rightarrow -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} =$

d)  $\lim_{x \rightarrow 0^-} e^{1/x} =$

e)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} =$

5) [10pt] Use the method of Ex 1 in Ch 2.1 (use a limit) to find the equation of the tangent

line to  $y = x^2$  at  $(-1,1)$ .

6) [15pts] Choose ONE of the problems below to do. Remember to use enough words and sentences - not just formulas. You can continue on the back.

a) Prove that  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$ .

b) Prove that there are two antipodal points on the equator with exactly the same temperature.

c) Prove that  $\lim_{x \rightarrow 2} \frac{x^2-4}{x-2} = 4$  using the  $\epsilon$  method. Hint: this function is ‘almost linear’, so you can use the normal approach with one small adjustment.

BONUS [5pts] Show that  $\lim_{x \rightarrow 3} x^2 = 9$  using the definition of limit.

**Remarks and Answers:** If you scored below 40/100 on this exam (and *many* students did), you have poor chances to pass this class. If you scored below 10 out of 25 on problem 1 (pre-calculus), you’d probably do better to re-take Algebra and/or Trig than to continue in Calculus I. Among the other students, the average was about 60/100. The unofficial scale is:

A’s = 75 to 100, B’s = 65 to 74, C’s = 55 to 64, D’s = 45 to 54, F’s = 00 to 44.

The scores were OK on the True-False and on the limit calculations (and much better than expected on the bonus), but rather bad on the pre-calculus, parametric eqns, tangent line and the proof.

1a)  $x > 0$ . You can solve this by algebra, or by graphing both sides. Trial and error is not very reliable.

1b)  $x = \pm\pi/4$  or  $\pm 3\pi/4$  or  $\pm 5\pi/4$ , etc. You can rephrase this to:  $x \in \{\pi/4 + n\pi/2 : n = 0, \pm 1, \pm 2 \dots\}$ . I gave partial credit if you only found one of the solutions, such as  $x = \pi/4$ , but of course trig equations like this usually have lots of answers.

1c) 999

1d)  $2 \sin(\alpha) \cos(\alpha)$ . This is identity (40) in the appendix; if you don’t know it, you can get it from identity (34) [which I told you to learn] by setting  $\beta = \alpha$ .

1e)  $\sqrt{1-x^2}$ . This is probably the only ‘hard’ part of exam problem 1, but it is homework problem 1.5.40a.

2)  $y = t$  and  $x = t^2$  with  $-1 \leq t \leq 1$ . [Do NOT use  $x = t$  and  $y = \sqrt{t}$  instead, because that implies  $y > 0$ , which is false]. This was homework problem 1.8.17.

3) TFFTFT Please ask about any you don't understand.

4)  $+\infty$ , 4, -1, 0,  $1/4$ . Most of these are exercises from Chs 2.2 and 2.3. Some of them are hard to explain precisely, so I didn't demand a lot, but you generally had to show some work for full credit. For part (a), ' $+\infty$ ' is a more specific answer than ' $\infty$ ' or 'd.n.e' (see the instructions for this problem). Of course, some other problems must be answered by 'd.n.e' (or by "diverges", which is the same).

5)  $y = -2x - 1$ . Most of the work is to find  $m_{tan} = -2$  using a limit of  $m_{sec}$  as done in class and in Ch 2.1. This was homework problem 2.1.17.

6a) and 6b) were done in class (and 6a is also in the book). For 6a), you really *need* a decent picture to explain the idea. For 6c), most people realized that  $f(x) = x + 2$  (unless  $x = 2$ ), so the proof is almost the same as for  $\lim_{x \rightarrow 2} x + 2 = 4$ . You can set  $\delta = \epsilon$ . You should also mention somewhere that since  $0 < |x - 2|$  (from the definition) we know  $x \neq 2$ , and can multiply on top and bottom by  $x - 2$ , etc.

Note 1: 6a was advertised - you should walk into each exam ready to write such proofs.

Note 2: Proofs are supposed to be convincing arguments (probably including calculations), which means they should be organized and explained clearly. Much of your grade was based on that.

Bonus: this (or very similar) was done in class. If you set  $\delta = \min(\epsilon/7, 1)$ , your calculations were probably right.