MAC 2311 Sept 25, 2007
Exam I Prof. S. Hudson

1) [10pt] Sketch the curve by eliminating the parameter: $x = t + 1$, $y = \sqrt{t}$, for $0 \leq t \leq 4$.

2) [10pt] Consider the function $g$ graphed in the accompanying figure. For what values of $x_0$, $-5 \leq x_0 \leq 5$, does $\lim_{x \to x_0} g(x)$ exist?

3) [10pt] Find a value for $k$ so that this function is continuous [so, $f(-2) = \lim_{x \to -2} f(x)$].

$$f(x) = \begin{cases} (x+2)/(x^2-4) & \text{if } x \neq -2 \\ 3/k & \text{if } x = -2 \end{cases}$$

4) (30pt) Compute the following limits. Notice that some are one-sided. You may have to answer, for example, with 'd.n.e.' or '$+\infty$' [the latter will get more credit, if both are correct].

a) $\lim_{x \to 0^+} \ln(x) =$

b) $\lim_{x \to +\infty} \sqrt{\frac{9x^2+16}{4x^2+25}} =$

c) $\lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} =$

d) $\lim_{x \to -\infty} 3e^{2x+1} =$

e) $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x} =$

f) $\lim_{x \to 1} \frac{x^3-1}{x-1} =$

5) [10pt] Use the method of Ex 1 in Ch 2.1 (use a limit) to find the equation of the tangent line to $y = 1/x$ at (2,1/2).

6) [20pts] Answer True or False. You do not have to explain.

a) If $f$ is continuous at 2, then $\lim_{x \to 2^+} f(x)$ exists.

b) If $\lim_{x \to 2^-} f(x)$ exists, then $\lim_{x \to 2^+} f(x)$ exists.

c) $f(x) = \frac{\sin(x)}{e^x}$ is continuous.

d) $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ is an identity.

e) $\forall \epsilon > 0, \exists \delta > 0, 3\delta < \epsilon$.

f) If $x$ is a number so that $|x-3| < 2$, then $|x-1| < 3$.

g) If $x$ is a number so that $|x-3|<1$, then $|x-1|<5$.

h) $\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$ is an identity.

i) $\forall\epsilon>0, \exists\delta>0, \delta+\epsilon<10$.

j) The equation $x^3+x^2-2x=1$ has at least one solution in the interval [-1,1].

7) [10-15pts] Choose ONE of the problems below to do. Remember to use enough words and sentences - not just formulas. You can continue on the back. Parts b) and c) are harder, so I will give +5 extra credit points if you choose one of those, and get it right.

a) Prove that $\lim_{x\to 0}\frac{\sin(x)}{x}=1$.

b) Prove that $\lim_{x\to 2} 2x^2 = 8$ using the definition of limit.

c) Prove that $\lim_{x\to 0^-}\frac{1}{x^2}=+\infty$ using the (revised) definition of limit.

**Remarks and Answers:** The average was about 70/100, which is pretty normal, so you can use the scale on the syllabus for now. On average, the results were fairly good on all the problems, except maybe 2) and 5).

1) $y=\sqrt{x-1}$ for $1\le x\le 5$. The graph is part of a parabola, only from (1,0) to (5,2). Your graphs should be labelled with these numbers (either on the points, or on the axes). A common mistake was to draw too much of the parabola, ignoring that $0\le t\le 4$. The formula $x=y^2+1$ is not as accurate, unless you state that $0\le y\le 2$.

2) All numbers, except $x_0=-3$ (and the endpoints - a technicality which I didn't count into the grading). You can use other notation, such as $(-5,-3)\cup(-3,5)$, but be careful to get it exactly right. This problem is essentially problem 2.1.5 from the textbook.

3) Set $3/k=\lim_{x\to -2}(x+2)/(x^2-4)=-1/4$ and get $k=-12$. See problem 2.2.34.

4a) $-\infty$ See page 130.

4b) 3/2

4c) -1. The $e^{-x}$ terms dominate, so you can cross out the $e^x$ terms, and then cancel.

4d) 0 See page 130.

4e) 1/4 Use the conjugate.

4f) 3 Factor, $x^3-1=(x-1)(x^2+x+1)$.

For parts 4c, 4e and 4f, I insisted on some work/reasoning, for full credit. The others are

relatively easy. Parts 4e and 4f are derivatives, though you don't need to know that to solve them. 4e is $\frac{d}{dx}\sqrt{x}|_{x=4}$ and 4f is $\frac{d}{dx}x^3|_{x=1}$.

5) $m_{sec} = (1/x - 1/2)/(x-2) = -1/(2x)$. Take a limit as $x \to 2$ and get $m_{tan} = -1/4$. So, the equation is $(y - 1/2) = (-1/4)(x-2)$. See also problems 2.1.17-20 (and the examples in Ch 3.1).

6) TTTTF FFTFT

7) See the text and lectures. This was a 10-point problem, but some people, who chose b) or c), got up to 15 points.

Most people chose b); one good choice of $\delta$ is $\delta = \min\{\epsilon/10, 1\}$, but others are possible. That's just one step of the proof, of course.