

1) (5pts each) Calculate the derivative of  $f$ :

a)  $f(x) = \sin(x^3)$

b)  $f(x) = x \ln(x) - x$

c)  $f(x) = \tan^{-1}(x) = \arctan(x)$

d)  $f(x) = |x + 1|$

e)  $f(x) = e^{-x}$

1f)  $f(x) = 2 \cot(x) - \sec(3x)$

1g)  $f(x) = \cos^3(\sin(2x))$

2a) (10pts) Calculate  $dy/dx$  using implicit differentiation for the Folium of Descartes:  $x^3 + y^3 = 3xy$ .

2b) (5pts) Use your answer to 2a) to find the equation of the tangent line to the Folium at the point  $(3/2, 3/2)$ .

3) (5pts) Find the second derivative,  $y''(x)$ , where  $y = (1 + x)/x$ .

4) (10pts) Compute  $y'$  using Log-Diff:  $y = x^x$ .

5) (10pts) Find the local linear approximation to  $\cos(x)$  at  $x_0 = \pi/6$  and use it to approximate  $\cos(31^\circ)$ . You can leave square roots and/or  $\pi$  (etc) in your answers.

6) (10pts) A 17 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5ft/sec, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground? [note:  $8^2 + 15^2 = 17^2$ ]

7) (5pts) Compute the limit (using LHR)

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2(x) + \tan(4x)}{5x} =$$

8) (10pts) CHOOSE ONE. Remember to explain everything;

A) Prove that the derivative of  $\cos(x)$  is  $-\sin(x)$  directly from the definition (use a limit, and explain any key steps).

B) State and prove the Product Rule.

C) Prove that there are two antipodal points on the equator with exactly the same temperature. [I may give a few extra credit points if you choose this one and get it right. But don't choose two proofs.]

Bonus (5pts) - I mentioned in class that  $y = e^x$  is a solution of a very simple differential equation. State it.

### Answers and remarks:

To help add up your grades, I wrote your combined score for 1a-1e in the margin of page one (likewise, for 1f-1g and 2a-2b on page 2).

1a)  $\cos(x^3) 3x^2$  (chain rule)

1b) Product Rule and simplification:  $\ln(x)$

1c) from memory,  $y' = 1/(1 + x^2)$

1d)  $y' = +1$  if  $x > -1$  and  $y' = -1$  if  $x < -1$ . (a graph may help answer this one)

1e)  $-e^{-x}$

1f)  $-2 \csc^2(x) - 3 \sec(3x) \tan(3x)$

1g) Chain Rule Practice:  $3 \cos^2(\sin(2x))(-\sin(\sin(2x))(\cos(2x)))2$ .

2a)  $(y - x^2)/(y^2 - x)$ . b) Plug-in to get  $m = -1$  and then  $y = 3 - x$

3) Simplify first!  $y = x^{-1} + 1$ ,  $y' = -x^{-2}$  and  $y'' = 2x^{-3}$ . You can use the Product Rule or Quotient Rule instead, but should still look for chances to simplify along the way. Since this problem should be easy, I took off one point for long messy answers.

4)  $y' = x^x(1 + \ln(x))$

5)  $\sqrt{3}/2 - \pi/360$

6)  $75/8$  ft/sec

7)  $4/5$

8B) See text. For full credit, you should use standard notation, include the hypotheses of the Theorem (that  $f$  and  $g$  are differentiable) and mention when you are using the definition of derivative, or continuity, or a limit theorem. I often let 1-2 of these slide, but if you didn't include *any* words, you lost a few points.

Fewer people chose A or C, but similar remarks apply.

Bonus:  $y' = y$ .