MAC 2311 Oct 25, 2006
Exam II and Key Prof. S. Hudson.

1) [10pts] Compute each limit (as in Ch 2.6 - do NOT use L'Hopital's Rule):

a) $\lim_{x\to+\infty} \ln(\frac{x+1}{x})$

b) $\lim_{x\to 0} \frac{3x^2}{1-\cos^2 x}$

2) [10pts] Compute the derivative of $f(x) = \sin(x)$ using the definition of derivative (eg compute a limit).

3) [5pts each] Compute y' using any valid method. You can answer one of these piece-wise, and can leave y in one answer, if necessary.

a) $y = \frac{3x}{2x+1}$

b) $y = \cos^4(x^3)$

c) $y = |x - 4|$

d) $y = x^7 \tan(x)$

e) $y = x \ln(x) - x$

f) $x^2 + y^2 = 1$

g) $y = \sqrt{1 + \sqrt{2 + x}}$

4) [15 pts] Answer True or False **and provide a brief explanation** for each answer.

If f is continuous at 4, then it is differentiable at 4.

$\tan(\frac{1}{10+x^2})$ is continuous.

The function $f(x) = \sqrt{1 - x^2}$ is defined implicitly by $x^2 + y^2 = 1$.

5) [10 pts] A man 6 ft tall walks with a speed of 5 ft/s away from a street light atop a 12-foot pole. How fast is the tip of his shadow moving along

the ground when he is 100 ft from the pole?

6) [10pts] Use a local linear approximation to estimate $\cos(43^o)$.

7) [10pts] CHOOSE ONE; (remember to explain properly)

A) State and prove the Power Rule.

B) State and prove the Product Rule.

BONUS (5 pts) Find the equations of two lines tangent to the circle $x^2 - 4x + y^2 + 3 = 0$ which pass through (0,0).

Remarks and Answers: The average was about 60/100, based on grades over 40. The unofficial scale is A's: 76-100, B's: 66-75, C's: 56-65, D's: 46-55. The average for Exam 1 is about the same, so you can use this scale for that, too, and for your semester average [which I wrote on the upper right corner of your Exam 2]. With hard work, you might expect to raise this one letter grade by the end of the semester. But raising an F to a C is not likely.

Generally, the worst scores occurred on problems 1, 4 and especially 5 (related rates). To encourage you to review these, I plan to put similar problems on Exam 3 and/or the Final.

1a) [see problem 2.6.20] Use the continuity theorem; ln $[\lim((x+1)/x] = \ln(1) = 0$

1b) [slightly revised 2.6.31] Use the trig identity; $3\lim[x/\sin(x)]^2 = 3$.

2) Done in class. Since the problem says "compute" (not "prove"), you didn't have to explain each step carefully. But every exam includes instructions to "show your work and reasoning" and you should make a special effort to do that here, since everyone knows the answer is $\cos(x)$. If you got any steps wrong, or missing, your explanation/organization probably affected your partial credit.

3) Everybody gets better at these calculations with practice. By the end of the semester, you should be getting at least 90% of them right. I wrote the scores for each question towards the right side of your page. The total

is on the left. Please check my addition.

3a) $3/(2x+1)^2$

3b) $4\cos^3(x^3)[-\sin(x^3)]3x^2$

3c) +1 if $x > 4$ and -1 if $x < 4$.

3d) $7x^6\tan(x) + x^7\sec^2(x)$

3e) $\ln(x)$

3f) $y' = -x/y$

3g) $(1/4)(1+(2+x)^{1/2})^{-1/2}(2+x)^{-1/2}$

4) Each correct T or F was worth 2 points. Each explanation was worth up to 3 points.

4a) False; $|x-4|$ is continuous at 4 but not differentiable at 4. Other explanations might be possible, but I cannot think of another simple one. It is true that every differentiable function is continuous, but this doesn't show that 4a is false (I gave +1 partial credit for saying that though).

4b) True: Since $1+x^2 \neq 0$, we know $\frac{1}{1+x^2}$ is continuous. Though $\tan(\theta)$ is not continuous at $\theta = \pm\ \pi/2 + n\pi$, it is impossible for $\frac{1}{1+x^2}$ to match any of these values, since $0 < \frac{1}{1+x^2} < 1 < \pi/2$.

4c) True: It is best to discuss the graphs (see page 236). I gave 4/5 points for 'True' with valid calculations and comments. But it is NOT true that the two equations are the same, and I deducted a point for saying that.

5) [slight revision of HW problem 3.7.32b, and done in class] 10ft/sec. About half the students drew a reasonable picture, but did not label variables (though it is not hard to do). Without doing that, there is almost no chance to succeed.

I usually gave about 6-7 points for an answer of 5ft/sec (= how fast the shadow length is increasing) but only if the work was clear enough.

6) $f(x) \sim f(x_0) + f'(x_0)\triangle x = \cos(45) - \sin(45)(-\pi/90) = (1+\pi/90)/\sqrt{2}$. Note that $\triangle x = 43 - 45 = -2$ must be converted to radians.

7) See the text or lecture notes. You should state the theorem clearly before proving it. These 2 proofs are mainly calculations, but you should

try to explain any step that is not just basic algebra. The proof of the binomial theorem should mention *the definition of derivative* and *the binomial theorem*. [and the Product Rule proof should mention continuity, and the definition of derivative three times].

Bonus: (done in class) $y = \pm x/\sqrt{3}$.