MAC 2311 March 1, 2006
Exam II and Key Prof. S. Hudson.

1) (5pts each) Calculate the derivative:

a) $f(x) = \sin(1/x^2)$

b) $f(x) = x^3 \sin(x)$

c) $f(x) = (2x+3)/(x+1)$

d) $f(x) = 2\cot(x) - \sec(3x)$

e) Find y'' when $y = x\sin(3x)$

2) (10pts) Use Ch 3.1 methods (*no derivatives!*) in this one. Let $y = 1/x$.

a) Find the average rate of change of y with respect to x over the interval [3,5].

b) Find the instantaneous rate of change of y with respect to x at 3.

3) (5pts) Find the discontinuities of $f(x) = \sec(x)$.

4) (10pts) Given that $f(4) = 3$ and $f'(4) = -5$, find $g'(4)$:

a) if $g(x) = xf(x)$

b) if $g(x) = f(16/x)$

5) (10pts) Find the local linear approximation to x^3 at $x_0 = 1$ and use it to approximate 1.02^3.

6) (10pts) A 10 ft ladder leans against a wall at an angle θ with the horizontal. The top of the ladder is x feet above the ground. If the bottom of the ladder is pushed towards the wall, find the rate at which x changes with respect to θ when $\theta = 60°$. Express the answer in units of feet/degree.

7) (10pt) A paint manufacturing company estimates that it can sell $g = f(p)$ gallons of paint at a price of p dollars. In practical terms, what does dg/dp mean in this case ?

What can you say about the sign of dg/dp ?

8) (10pts) Answer with True or False:

If f is continuous it is differentiable.

Acceleration is the derivative of velocity.

If $f'(x) > 0$ for all x then $f(x) > 0$ for all x.

If g is differentiable, then $(1/g)' = g'/g^2$.

$|\sin(x)|$ is differentiable at $x = \pi$.

9) (10pts) CHOOSE ONE. Remember to explain everything;

A) Prove that the derivative of $\cos(x)$ is $-\sin(x)$ directly from the definition (use a limit, and explain any key steps).

B) State and prove the Power Rule (for $n = 1, 2, 3 \ldots$).

Bonus (5pts) - I mentioned that $y = \sin(5x)$ solves a simple differential eq'n. State it.

Remarks and Answers: The average among the top 20 scores was approx 63, so the unofficial scale for this exam is about 5 points lower than the one on the syllabus. The results were pretty good on problems 1 and 9; pretty bad on problems 3 (See 2.6.4) and 7 (See 3.2.36). Most of these problems came directly from the textbook.

1a) $-2\cos(1/x^2)/x^3$

1b) $3x^2\sin(x) + x^3\cos(x)$

1c) $-1/(x+1)^2$ If you used the Quotient Rule on this one, you should simplify the answer because it is so easy to do (but I didn't take off points this time). You can also solve this one easily using $2x + 3 = 2(x + 1) + 1$, to simplify the fraction before taking d/dx.

1d) $-2\csc^2(x) - 3\sec(3x)\tan(3x)$

1e) $y' = \sin(3x) + 3x\cos(3x)$ and $y'' = 6\cos(3x) - 9x\sin(3x)$

2a) $(1/5 - 1/3)/(5 - 3) = -1/15$. 2b) Use a limit to get -1/9.

3) It is discontinuous where $\cos(x) = 0$. For example, at $x = \pi/2$ or $x = -15\pi/2$. Some people wrote "at odd multiples of $\pi/2$", which is OK. For a more standard answer, write

$$x = \pi/2 \pm n\pi \text{ for } n = 0, 1, 2 \ldots$$

4a) -17 (Product Rule) and 4b) +5 (Chain Rule)

5) A general form of the LLA is $f(x_0) + f'(x_0)\triangle x$.
Using $f(x) = x^3$ and $x_0 = 1$, the LLA for this problem is $1 + 3(x - 1)$.
Then set $x = 1.02$ to get $1.02^3 \approx 1.06$.
Note: the answer to 5a) is in line 2 above [not line 1].

6) As soon as you see the θ you can expect that the constraint equation involves trig (not Pythagoras). Start with $\sin(\theta) = x/10$, so $\cos(\theta) = \frac{dx}{d\theta}\,/10$. So, $\frac{dx}{d\theta} = 5$ ft/rad $= 5\pi/180$ ft/deg. When you take a derivative of a trig function like this, the angle is in radians. So, the '5' is in ft/rad. Multiply by π rad/180 deg to change the units to ft/deg.

7a) It is the change in sales [in gallons] resulting from an increase in price. The units are gallons per dollar 7b) Negative; higher prices mean lower sales.

8) FTFFF

9) See the text or lecture notes. Both of these are essentially calculations, but for full credit, you must explain with phrases like "by the definition of derivative".

Bonus: $y'' = -25y$