MAC 2311 — Oct 18, 2007

Exam II and Key — Prof. S. Hudson.

1) (5pts each) Calculate the derivative:

a) $f(x) = \sin^3(x)$

b) $f(x) = \csc(x) + x\cot(x)$

c) $f(x) = \ln(x)/(x+1)$

d) Find y' using implicit differentiation, given that $\sin(xy) = y$.

e) Find y'' when $y = x\sin(3x)$

2) (10pts) Use basic Ch 3.1 methods, such as division and/or limits, for this one. *No shortcuts involving derivatives!*. Let $y = 1/x$.

a) Find the average rate of change of y with respect to x over the interval [3,5].

b) Find the instantaneous rate of change of y with respect to x at 3.

3) (5pts) Find the discontinuities of $f(x) = \tan(2x)$ on the interval $0 \leq x \leq 2\pi$.

4) (10pts) Given that $f(4) = 3$ and $f'(4) = -5$, find $g'(4)$ in each case. These are two separate problems:

a) if $g(x) = xf(x)$

b) if $g(x) = f(16/x)$

5) (10pts) Find the local linear approximation to $\sqrt{x}$ at $x_0 = 1$ and use it to approximate $\sqrt{1.02}$.

6) (10pts) A 13 ft ladder leans against a wall. Suppose the bottom of the ladder is pushed towards the wall at a rate of 2 ft per sec. Find the rate at which the top of the ladder is rising, when the base is 5 ft from the wall.

7) (10pt) Consider the position vs time curve below, for a car moving along a straight road [if it is missing, or unclear, please let me know]. It shows that the car has gone 2.5 meters after 5 seconds. Use the line segment in the graph to estimate the instantaneous velocity of the car at that instant.

8) (10pts) Answer with True or False:

If f is differentiable it is continuous.

Every secant line for $f(x) = \ln(x)$ has a positive slope.

The function $f(x) = \cos(x)\sec(x)$ has a removable discontinuity at $x = \pi/2$.

If g is differentiable, then $(1/g)' = -g'/g^2$.

$|\sin(x)|$ is differentiable at $x = \pi$.

9) (10pts) CHOOSE ONE. Remember to explain everything;

A) Prove that the derivative of $\cos(x)$ is $-\sin(x)$ directly from the definition (use a limit, and explain any key steps).

B) State and prove the Power Rule (for $n = 1, 2, 3 \ldots$).

Bonus (5pts) - Find the equation of a tangent line to $y = \sqrt{x-2}$ which passes through the origin.

Remarks and Answers: The average grade was about 70. You can use the original scale (on the syllabus) for this exam. A's = 81-100, etc. The average grade was pretty good on every problem, except the 5-point problem 3, and perhaps problems 4 and 7. There were 4 grades in the 90's, but also many F's.

Your semester average so far is Avg = (PreQuiz + 3*Exam1 + 4*Exam2)/8. I wrote your letter grade in red on your exam, using the same scale as above. Your HW and two more exams will be factored in later, of course.

If you see a problem graded 3^+, for example, it counts the same as a 3. This is just a note to myself that I wanted to give you a little more than 3 (but not 4). I may have 'rounded up' on some later problem as a result.

1a) $3\sin^2(x)\cos(x)$

1b) $-\csc(x)\cot(x) + \cot(x) - x\csc^2(x)$

1c) $[(x+1)/x - \ln(x)]/(x+1)^2$

1d) $y' = (y\cos(xy)) \ / \ (1 - x\cos(xy))$

1e) $y' = \sin(3x) + 3x\cos(3x), \quad y'' = 6\cos(3x) - 9x\sin(3x)$

2a) $-1/15$

2b) $-1/9$

3) Set $\cos(2x) = 0$, so $2x = \pi/2$ or $3\pi/2$, etc. So, $x = \pi/4,\ 3\pi/4,\ 5\pi/4$ or $7\pi/4$.

For many functions (like this one) a discontinuity is just a value of x that you can't plug-in [but this is not the precise definition, of course]. Many people thought $x = \pi/2$

was a discontinuity of f, but you *can* plug that in; $f(\pi/2) = \tan(\pi) = 0$, which is a big clue to revise that answer! See problems 2.6.1-14 for more practice.

4a) $3 - 20 = -17$ Product Rule (See problems 3.4.19-22)

4b) $(-5)(-1) = 5$ Chain Rule; no partial credit for -5. A sneaky alternative for both 4a and 4b is to set $f(x) = 3 + (-5)(x - 4)$ (from the LLA formula), but that's a bit more work.

5) The LLA is $g(x) = 1 + (1/2)(x - 1)$. Of course, you didn't get full credit if you ignored this aspect of the problem. The formula should include an 'x', but not an 'f' (which appears only in the general form of an LLA), and I prefer $(x - 1)$ over $\triangle x$ in this formula.

The approximation is $g(1.02) = 1.01$. In case of minor errors, I gave more partial credit for plausible answers like 1.001, than ones like 0.90 or 3.

6) 5/6 ft/sec. Most people either set this up carefully, and got it right, or they got nowhere. As usual, the first step is to *label the variables* clearly. People who didn't do this often got their x's and y's confused later on - in addition to confusing the poor grader. See 3.7.17(etc).

7) I get $v(5) = 1$, but I accepted any estimate within about 30 percent of that as reasonable, and gave full credit, if you showed your work. The plan is $v(5) = m_{tan} =$ rise/run, but you need two points on the tangent line to finish. I chose (2.5,0) and (5,2.5), for example, to get rise/run = 2.5/2.5 = 1. If you pick the two points very close together, you may lose some accuracy (but I didn't mark off for that). See problems 3.1.1 - 4 (etc).

8) TTTTF

9) See text/lectures. These don't require a lot of explanation, but I'd expect you to mention the definition of derivative, for example.

Bonus: $y = \sqrt{2}\ x/4$. I'd strongly recommend drawing a picture, but it's hard to do in the answer key. The calculation goes: $f'(x) = 1/(2\sqrt{x-2}) = m_{tan} = y/x = \sqrt{x-2}/x$. Algebra gives $x = 4$ and $y = \sqrt{2}$ and $m_{tan} = \sqrt{2}/4$. The equation is $y = mx$, so $y = \sqrt{2}\ x/4$. See also 3.4.27 (etc).