MAC 2311 November 21, 2005
Exam III Prof. S. Hudson

1) (15 pts) Use the info in these number lines [which were a bit clearer on the real exam] to describe the intervals of inc/dec and concavity for the function $f(x) = e^{-x^2/2}$

$$+ + + + + + + + + + + + + (0) - - - - - - - - - - - - - - \quad f'$$

$$+ + + + + + (-1) - - - - - - (+1) + + + + + + \quad f''$$

2) (15 pts) Analyze and graph $y = x^4 + 2x^3 - 1$. Plot and label any critical points and inflection points.

3) (10pts) Use both the First and Second Derivative Tests to show that $3x^2 - 6x + 1$ has a relative minimum at $x = 1$.

4) (15pts) Compute the limits:

a) $\lim_{x\to\pi} \frac{\sin(x)}{\pi - x}$

b) $\lim_{x\to 0} \csc(x) - 1/x$

c) $\lim_{x\to\infty}(1 - 3/x)^x$

5) (15 pts) Mark each sentence True or False;

$\sec(x)$ has a maximum value on the interval $(-\pi/2, \pi/2)$.

$\sec(x)$ has a maximum value on the interval $(\pi/2, 3\pi/2)$.

If c is a critical point of f, then f has a relative extrema at c.

A rational function can have 4 vertical asymptotes.

If a polynomial f has 3 different roots, it must have at least 2 critical points.

6) (15pts) Compute the derivative, y':

a) $y = \sin^{-1}(x) + \cos^{-1}(x)$ (and simplify the answer)

b) $y = (\ln x)^{\tan x}$

c) $x^3 + x \tan^{-1} y = e^y$

7) (10pts) Find equations for the asymptotes of $y = 3x^2/(x^2 - 4)$

Answers and Remarks: We went over a lot of the exam on 11/23/05, so I will be brief (if you want help, ask!). The average was about 72, with two high scores of 95. Adjust the original scale upwards about 3 points for this exam (for example, an 83 is a B+, but an 84 is an A-). By mistake, the exam had only 95 points, so everybody got 5 points of extra credit at the end.

The scores were good, except on the direct calculation problems, 4 and 6. These should be straightforward with enough practice.

1) The function is increasing on $(-\infty, 0)$ and is decreasing on $(0, +\infty)$. It is concave up on $(-\infty, -1)$ and on $(1, +\infty)$. It is concave down on $(-1, 1)$.

The phrase "and on" should not be omitted, or replaced by $\cup$, or "and at" (which is used with points, but not intervals). These errors are a bit more serious than grammar mistakes and typically lost one point.

2) The stationary points are at (-3/2, -43/16) and (0,-1). The inflection points are at (-1,-2) and (0,-1). The graph looks like a parabola on the sides, but has a bump (concave up) where $-1 < x < 0$. "Analyze" means you should also discuss the intervals, but I gave credit if this info was clear from a number line, for example. If you made an early mistake (eg, finding f' or the +/- pattern), it is unlikely your graph was accurate enough to get many points. You just have to be extra careful with these. But I did not take off points, if you got -43/16 slightly wrong.

Notice that the phrase "are at" is OK for this problem, because it is followed by points, not intervals.

3) After the calculations, say something like this - "Since $f' < 0$ on $(-\infty, 1)$ and $f' > 0$ on $(1, +\infty)$, the First Derivative Test implies that f has a relative minimum at $x = 1$." [And for the Second D.T., mention that $f''(1) > 0$].

NOTE: The phrase "show that" implies you must explain your reasoning in words, at least briefly. I did not give full credit for answers like "$f''(x) = 6$", or "f is a parabola". You can talk about where f is increasing, concave up (etc), but you don't have to. These ideas are not explicitly in the Tests.

Also, these two tests are completely separate from each other, so you should give two separate answers (some people tried to combine them into a single explanation).

4a) 1 (after using LHR)

4b) 0 (after LHR 3 times)

4c) e^{-3}

5) FTFTT

6a) $y' = 0$

6b) Use Log-Diff: $y' = (\ln x)^{\tan x}[\frac{\tan x}{x \ \ln x} + \sec^2 x \ \ln(\ln x)]$

6c) Use Implicit Diff: $y' = \frac{3x^2 + \tan^{-1} y}{e^y - x/(1+y^2)}$

7) VA's at $x = 2$ and $x = -2$. HA at $y = 3$. Note that the question asks for the *equations* of the lines, so an answer like "3" gets less credit than "$y = 3$".