MAC 2311 | April 4, 2005
Exam III | Prof. S. Hudson

Name

Show all your work and reasoning for maximum credit. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. Hand in any extra paper you use along with your exam.

1) (15 pts) Analyze and graph $y = x^4 + 2x^3 - 1$. Remember to
show all your work
describe the intervals of inc/dec, concavity
plot and label any critical points and inflection points.

2) (10 pts) Show that $f(x) = 2\sin(x)\cos(x)$ satisfies the hypotheses of Rolle's theorem on $[0, \pi]$, and find all numbers x that satisfy the conclusion of that theorem. [You can use the letter c instead of x if you like].

3) (20pts) Answer True or False:

sec(x) has a maximum value on the interval $(-\pi/2, \pi/2)$.

The graph of every cubic polynomial has exactly one point of inflection.

If c is a critical point of f, then f has a relative extrema at c.

A rational function can have 4 vertical asymptotes.

If a polynomial f has 3 different roots, it must have at least 2 critical points.

If f is decreasing on I, then $f'' < 0$ there.

The maximum value of $2\sin(x) + 2\cos(x)$ is 4.

If f has only one relative extrema on I, and it is a relative minima, then it is also an absolute minima.

If f is continuous on (a,b) and $\lim f = -\infty$ as $x \to a^+$ and as $x \to b^-$ then f has an absolute max but no absolute min on (a,b).

If $f\,''(3) = 0$ then f has an inflection point at 3.

4) (15 pts) a) Find and classify the critical points (as rel max's or rel min's) given that the *derivative* is $f\,'(x) = x^3(x^2 - 4)$.

b) Use the second derivative test to classify the critical points of $f(x) = \sin^2(x)$ on $0 < x < 2\pi$.

c) Use the first derivative test to classify the critical points of $f(x) = x^{4/5}$.

5) (15 pts) Compute each limit:

a) $\lim_{x\to 1} \frac{\ln(x)}{x-1} =$

b)$\lim_{x\to 0} \frac{e^x+e^{-x}-2}{x\sin(x)} =$

c) $\lim_{x\to 0} \frac{1-\cos(x)}{\cos(2x)}$

6) (15pts) Suppose the rocket in the figure is rising at a rate of 880 ft/s when it is 4000 ft up. How fast must the camera elevation angle change to keep the rocket in sight? Answer in radians per second. [If you don't understand the story or the question, you can ask me about it].

The exam included a picture of the rocket, showing the launch pad 3000 ft from the camera.

7) (10 pts) Choose ONE proof. Remember: for full credit, *explain carefully.*

A) State and prove Rolle's theorem (state Case 2, but don't prove it).

B) If $f'(x) < 0$ on (a,b), then f is decreasing there.

Bonus (5pts) - A rational function can have a *slant* asymptote of the form $y = mx + b$ if $f(x) - (mx + b) \to 0$ as $x \to \infty$. Find the slant asymptote of $f(x) = \frac{x^3}{x^2+1}$ by doing a long division. Explain your answer briefly.

Answers: After removing some very low scores, the average was about 60. The unofficial scale has A's starting at 73, B's at 63, etc. Again, the lowest scores occurred on the related rates problem. We will practice more word problems in class, and try this again on the Final Exam.

1) The critical points are -3/2 and 0. The function decreases on $(-\infty, -3/2)$ and increases on $(-3/2, \infty)$ [if you split the last interval up at 0, I gave also full credit).

The inflection points are at -1 and 0. The function is concave up on $(-\infty, -1)$ and on $(0, \infty)$. It is concave down on $(-1, 0)$.

The graph is roughly "U" shaped, with a min at $x = -3/2$.

2) Check that $f(0) = f(2\pi) = 0$ and mention that f is "nice". You might note that $f(x) = \sin(2x)$ (though you can work the problem without this, of course). Then set $0 = f'(x) = 2\cos(2x)$ and get $x = \pi/4$ or $3\pi/4$.

3) FTFTT FFTTF

4a) The cr.pts are -2,0,2 with the pattern -,+,-,+ [dec, inc, dec, inc]. So, -2 and +2 are rel mins, while 0 is a rel max.

4b) $f'(x) = 2\sin(x)\cos(x) = \sin(2x)$, which is 0 when $x = \pi/2,\ \pi,\ 3\pi/2$. Plugging into $f''(x) = 2\cos(2x)$ we get -,+,-, so these are rel max, min, max in that order.

4c) $f'(x) = 0.8x^{-1/5}$ which is n.d. at $x = 0$. And f' goes -,+ there. So this is a rel min.

5) 1, 1, 0 [but very little credit without correct work].

6) As usual, start by assigning variables (e.g., height = y). Then $\tan\theta = y/3000$, so $[\sec^2\theta]\ \theta' = y'/3000 = 880/3000$. Set $\sec\theta = 5/3$ and get $\theta' = (3/5)^2(880/3000)$ rad/sec.

7) See the text. Of course, you need to prep for these before you come to the exam.

Bonus) Use long division and get $f = x - x/(x^2+1)$. So, the asymptote is $y = x$.