MAC 2311 November 22, 2006
Exam III and Key Prof. S. Hudson

1) (15 pts) Use the info provided to analyze and graph the function $f(x) = \frac{x^2-2x+4}{x-2}$. [Draw the f' and f'' lines and discuss intervals. Find all the HA's, VA's, critical points and inflection points - if they exist].

Given: $f'(x) = \frac{x(x-4)}{(x-2)^2}$ and $f''(x) = \frac{8}{(x-2)^3}$. Also, notice that $x^2 - 2x + 4 > 0$ for all x.

2) (15 pts) Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of the circular opening (at the top) is 6 feet, how fast is the water level rising when the water is 4 feet deep? Hint: $V = \pi r^2 h/3$.

3) (15pts) Compute the limits:

a) $\lim_{x\to\pi} \frac{\sin(x)}{\pi - x}$

b) $\lim_{x\to 1^+} (\frac{x}{x-1} - \frac{1}{\ln x})$

c) $\lim_{x\to 0} (\cos x)^{1/x}$

4) (10pts) State and prove Rolle's Theorem (prove Cases 1 and 3 carefully, if not Case 2).

5) (15 pts) Mark each sentence True or False;

$\tan(x)$ has a maximum value on the interval $(0, \pi)$.

$f(x) = |x^2 - 4| + 1$ satisfies the hypotheses of the Mean Value Thm on $[-4, 1]$.

If f has a relative extrema at c, then c is a critical point of f.

If $f(x)$ is decreasing, then it has an inverse function.

$f(x) = |x^2 - 4|$ satisfies the hypotheses of Rolle's Thm on $[-2, 2]$.

6) (15pts) Compute the derivative, y'. Do not leave y in your answer.

a) $y = \sin^{-1}(2x)$

b) $y = (\ln x)^{\tan x}$

c) $e^{x+y} = x + y + 4$ (use Imp.Diff)

7) (10pts) Find the absolute extrema of $f(x) = x^3 - 3x + 1$ on $[-3/2, 3]$.

Answers: The average for Exam 3 is in the high 60's. So is the average of all three exams. You can use the scale on the syllabus for both. I am writing this key partly from memory, so there could be errors. Please let me know if you spot any!

1) [As of 11/28/06, the graph for problem 1 is posted as a separate link on the Exam page]. The graph has a VA at $x = 2$ (put $x = 2$ on both number lines). No HA; the right side of your graph should approach infinity in the first quadrant (ideally at about a 45 degree angle). It should not look like a second VA there. Same idea for the third quadrant.

Critical (stationary) points at 0 and 4. No inflection points.

It increases on $(-\infty, 0)$ and on $(4, \infty)$.

It decreases on $(0, 2)$ and on $(2, 4)$, which is NOT the same as saying it decreases on $(0, 4)$. That's false, since $f(0) < f(4)$, for example.

It is concave up on $(2, \infty)$ and concave down on $(-\infty, 2)$.

I often gave one point extra credit for other info (not required) such as: no x-int, y-int at -2 and an oblique asymptote at $y = x - 2$.

2) Draw a picture. The variables are V, h, r, and t. You are given $V = \pi r^2 h/3$ and $dV/dt = 8$. You are asked for dh/dt. It's a good idea to get rid of r as soon as possible. The picture shows that r is proportional to h, so $r = c \cdot h$ for some number c. At the top of the tank, $r = 6$ and $h = 12$, so $c = r/h = 1/2$. So, $r = h/2$. [You can also get this formula from the picture and the idea of 'similar triangles']. Now plug it into the constraint (to remove the r) and get $V = \pi h^3/12$.

Now it's easy to apply d/dt, and get $dV/dt = (\pi h^2/4)dh/dt$. Plug in $h = 4$ and $dV/dt = 8$, and get $dh/dt = 2/\pi$ feet per minute.

3) 1, 1/2, 1

4) See the text.

5) FFTTT

6) a) $\frac{2}{\sqrt{1-(2x)^2}}$

b) $y' = (\ln x)^{\tan x}(\sec^2 x \;\ln(\ln(x)) + \frac{\tan x}{x \ln x})$

c) $e^{x+y}(1+y') = 1+y'$, so $(e^{x+y}-1)(1+y') = 0$, so either $y' = -1$ or $x+y=0$. So, in either case, $y' = -1$.

7) The critical points are -1 and +1. We get $f(-3/2) = 2\frac{1}{8}$ I meant to give you this one, but forgot. Sorry) $f(-1) = 3$, $f(1) = -1$ and $f(3) = 19$. So the max is 19, at 3, and the min is -1, at 1.