MAC 2311 April 3, 2006
Exam III and Key Prof. S. Hudson

1) (10pt) Analyze the function $f(x) = 3x^4 - 4x^3$. Find all the special points, and the intervals where it is increasing, concave up (etc).

2) (10pt) Sketch a graph of $y = (2x^2 - 8)/(x^2 - 16)$ including all asymptotes. Some hints: The function is even. It has x-intercepts at -2 and +2. The derivative is $y' = -48x/(x^2 - 16)^2$. Also, $y'' = 48(16 + 3x^2)/(x^2 - 16)^3$ is never zero. $f(0) = 1/2$ and $f(5) = 14/3$.

3) (5pt each) Compute the derivative of each:

a) $y = \ln(\frac{x}{1+x^2})$

b) $y = (\frac{x-1}{x+1})^{1/5}$ (use log-diff)

c) $y = \cot^{-1}(\sqrt{x})$.

d) $y = 2^x$

4) (10pts) A 13-ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft per second, how fast will the foot be moving away from the wall when the top is 5 ft above ground ?

5) (15pts) Answer True or False:

If $f'' > 0$ on [3,4] then f is concave up there.

If $f''(6) = 0$ then $x = 6$ is an inflection point of f.

If f is defined on the closed interval [3,4] then it has a maximum value there.

If $f'(x) = \ln(x)$ on [0.5,2.5] then f has an inverse function.

If $f'(0) = 4$ and $f''(x) > 0$ on [0,4], then f is increasing on [0,4].

6) (5pts each) Compute the limit.

a) $\lim_{x\to 0} \frac{e^x - 1}{\sin(x)}$

b) $\lim_{x\to\infty} (\frac{x+1}{x-1})^x$

c) $\lim_{x\to 0} \tan(x)\ \ln(x)$

7) [10pts] Choose ONE of these.

a) Compute the derivative of $\sin^{-1}(x)$ using the formula for the the derivative of an inverse function, and simplify the answer using a triangle.

b) Find the maximum and minimum values of $f(x) = 2x^3 + 3x^2 - 12x$ on [-3,2].

8) (10pts) Interpret $\lim_{h\to 0} \frac{10^h - 1}{h}$ as a derivative. Then compute the derivative to compute the limit. [Of course, you may check your answer by computing the limit some other way].

Bonus [5pts]: Show that the rate of change of $y = 3^{2x}5^{7x}$ is proportional to y.

Remarks and Answers: The average was about 59/100. The scores were good on most problems, especially 7). The worst scores were on problem 8). Also, problems 5) (True-False!!) and 6) had rather low scores.

1) It is decreasing on $(-\infty,\ 1]$ and increasing on $[1,\ \infty)$. Both $x = 0$ and $x = 1$ are stationary points. It is concave up on $(-\infty,\ 0)$ and on $(2/3,\ \infty)$. It is concave down on $(0,\ 2/3)$ Both $x = 0$ and $x = 2/3$ are inflection points. [See 5.1.15].

2) HA at $y = 2$. VA at $x = -4$ and $x = 4$. This is Example 1 from Ch 5.3. You can find the work and the graph there.

3a) Start with $y = \ln x - \ln(1 + x^2)$ and get $y' = 1/x - 2x/(x^2 + 1)$.

3b) $y' = (\frac{x-1}{x+1})^{1/5}[|x - 1|^{-1} - |x + 1|^{-1}]/5$.

3c) $-1/[2\sqrt{x}(1 + x)]$

3d) $(\ln 2)2^x$

4) Draw a picture and label it so I know what you mean by x or y or h or whatever letters you choose. The answer is 5/6 ft/sec.

5) TFFFT

6a) 1, 6b) e^2, 6c) 0 [credit here only when the method is correct]

7) Most people chose b) and got it right. Min of -7 (at $x = 1$), Max of 20 (at $x = -2$).

8) Write down the definition of derivative, and match it with this problem by setting $f(x) = 10^x$ and $x = 0$. The answer is $f'(0) = \ln 10$.
This is exercise 4.3.67, very similar to exercise 4.3.68, which was *an assigned HW problem.* Be prepared for problems like this! If you have trouble with a HW problem, ask!

Bonus: Compute y' using log-diff and show that $y' = Cy$ where C is a constant.