MAC 2311 Dec 13, 2006
Final Exam and Key Prof. S. Hudson

1) (20 pts) Compute and simplify. Include an arbitrary constant, if appropriate;

$\int \frac{3x^2}{1+x^3}\ dx =$

$\int \frac{3}{1+x^2}\ dx =$

$\int \sin^2 t\ dt =$

$\int e^{3x+2}\ dx =$

2) (20 pts) Compute;

$\frac{d}{dx} x^x$

$\frac{d}{dx}(\cos 2x\ \tan 2x)$

$\frac{d}{dx} \log_2 x^3$

Find dy/dx, given that $xy^2 + x^2y = y$.

3) (10 pts) Solve this initial-value problem; find a formula for $y(t)$ so that $y'(t) = 6(t+1)^2$ and $y(0) = 7$.

4) (10 pts) Compute;

$\lim_{x\to 0^+}(1+3x)^{1/x}$

$\lim_{x\to 1} \frac{\ln(x)}{\sin(\pi x)}$

5) (10 pts) Answer TRUE or FALSE:

The slope of the graph of $\tan^{-1}(x)$ is positive for all x.

The domain of $\sin^{-1}(x)$ is $[-1, 1]$.

Every polynomial has exactly one antiderivative whose graph contains the point (3,4).

The function $f(x) = \sec(2x)$ has an inverse on $[0, \pi/6]$.

If F is an antiderivative of an antiderivative of f, then $F\,''(x) = f(x)$.

6) (10 pts) Given parametric equations, $x(t) = t^2 + 1$ and $y(t) = t^3 + 1$, find the equation of the tangent line at the point where $t = 2$.

7a) (5pts) State the Intermediate Value Theorem.

7b) (5pts) State the Extreme Value Theorem.

8) (10 pts) CHOOSE ONE;

A) Prove the Product Rule.

B) Prove (as in Ch2) that $\lim_{x\to 0} \frac{\sin x}{x} = 1$

C) Prove (using ϵ) that $\lim_{x\to 0} 4x + 7 = 7$.

Bonus (5pts): Use Newton's method to approximate $\sqrt{3}$ (a positive root of $x^2 - 3 = 0$), starting from $x_1 = 2$. Just do one step (stop at x_2).

Remarks and Answers: Average = 63/100

1a) $\ln|1 + x^3| + C$. Minus 1 point for $\ln(1 + x^3) + C$ or just $\ln|1 + x^3|$.

1b) $3\tan^{-1} x + C$

1c) $\int (1 - \cos 2t)/2 \; dt = t/2 - (\sin 2t)/4 + C$

1d) $e^{3x+2}/3 + C$

2a) $x^x(1 + \ln x)$ (use log-diffn)

2b) Simplify first! $\frac{d}{dx} \sin 2x = 2\cos 2x$. If you applied the Product Rule correctly, but did not simplify the answer, you got 4 out of 5 points.

2c) $3/(x \ln 2)$

2d) $y' = (y^2 + 2xy)/(1 - 2xy - x^2)$ from Imp Diffn. Or, you can simplify first, and get $y = 1/x - x$ (or $y = 0$) so $y' = -1/x^2 - 1$ (or $y' = 0$). The few people who used the second method all forgot to include the possibility $y' = 0$, but I thought this approach was creative, so I gave full credit anyway.

3) $y = 2(t + 1)^3 + 5$

4a) e^3

4b) $-1/\pi$

5) TTTTT

6) $y = 3x - 6$ or $(y - 9) = 3(x - 5)$.

7) See the text. In any math course, focus on the definitions and theorems, especially theorems *with names.*

8) See the text.

Bonus) $2 - 1/4 = 1.75$