MAC 2311 Dec 12, 2005
Final Exam A [recent topics] and Key Prof. S. Hudson

1) (20 pts) Compute and simplify;

$\int x^7 + e^x \ dx =$

$\int \sec(x)\tan(x) \ dx =$

$\int \frac{2x}{x^2+1} \ dx =$

$\int \frac{1-2t^3}{t^3} \ dt =$

2) (15 pts) More:

$\int e^{5x} \ dx =$

$\int \cos^2(x) \ dx =$

$\int \tan^2(x) \ dx =$

3) (15 pts) A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (I drew a picture on the board - imagine a rectangle divided down the middle by a vertical line). What dimensions should be used so that the enclosed area will be a maximum? [If you don't understand this story, ask me!]

4) (15 pts) Sketch a graph of $y = \frac{x^2-1}{x^3}$. Find all critical points, inflection points and asymptotes [and label them clearly]. You may use:

$y' = \frac{3-x^2}{x^4}$
$y'' = \frac{2(x^2-6)}{x^5}$
$\sqrt{3} \approx 1.8$
$\sqrt{6} \approx 2.4$
$1.8^{-3} \approx 0.18$
$2.4^{-3} \approx 0.07$

5) (10 pts) CHOOSE ONE;

A) State and prove Rolle's thm.

B) Prove that if $f'(x) > 0$ on [a,b] then f is increasing on [a,b].

C) Prove the substitution rule of ch 6.3.

6) (15 pts) Answer TRUE or FALSE:

A continuous function defined on $(-\infty, +\infty)$ must have a minimum value.

If f is differentiable on the open interval (a, b) then it is continuous on $[a, b]$.

If f is a polynomial, then it has exactly one antiderivative whose graph contains the origin.

If $f(2) = f(3)$ then there is a point $c \in (2, 3)$ where $f'(c) = 0$.

If F is an antiderivative of an antiderivative of f, then $F''(x) = f(x)$.

7) (10 pts) Find the maximum and minimum values of $f(x) = |6 - 4x|$ on the interval [-3,3].

Extra Credit 1 [5pts]: Use the integration process of Ch 6.4 [not geometry or antiderivatives] to find the area under $y = 1 + 3x$ and above the interval $0 \leq x \leq 2$. You can answer on the back, but leave me a note here!

Extra Credit 2 [3pts]: Approximate $\sqrt{3}$ using Newton's method. You must decide what $f(x)$ should be for this problem! Start with $x_0 = 2$ and do two iterations. You can use these, but show all your other work:

$2 + 1/4 = 2.25$
$(2.25^2 - 3)/4.5 \approx 0.458$
$(1.75^2 - 3)/3.5 \approx 0.018$
$(1.5^2 - 3)/3 = -0.25$

Answers + Remarks The average was about 63, with relatively little variation (not many A's or F's). The grades were OK on all the problems, except for a 6/15 average on problem 2 (the slightly trickier antiderivative problems). Maybe problem 4 too.

1a) $x^8/8 + e^x + C$

1b) $\sec x + C$

1c) $\ln(x^2 + 1) + C$

1d) $-t^{-2}/2 - 2t + C$

2a) $e^{5x}/5 + C$ (set $u = 5x$)

2b) $x/2 + \sin(2x)/4 + C$ (use $\cos^x = (1 + \cos(2x))/2$

2c) $\tan^2 x - x + C$ (use $\tan^2 x = \sec^2 x - 1$).

3) Width = 50 ft, Height = 100/3 = 33.3 ft.

4) See Ch 5.3, Example 2.

5) See the text. For A, I wanted you to mention the Extreme Value Theorem, why the max is not at an endpoint, state the 3 cases clearly, etc. For B, mention the MVT, definition of increasing, etc.

6) FFTFT

7) Max of 18 when x = -3. Min of 0 when x = 3/2.

EC1) This is similar to Ch 6.4, Example 7. The final answer is 8 (but I mainly graded the method).

EC2) $x_1 = 1.75$ and $x_2 = 1.75 - 0.018$