

Show all your work. Use the space provided, or leave a note. Don't use a calculator or your own extra paper. Problems 3-8 are worth 10 points each.

- 1) [5 points each] Short calculations. Simplify any easy terms like  $\sin(\pi)$  and  $e^0$ .

$$\sum_{k=2}^{20} \frac{1}{k} - \frac{1}{k-1} =$$

$$\sum_{k=0}^{20} 2^k = \text{(Hint: } S - rS = a - ar^{n+1}\text{)}.$$

$$\frac{d}{dx} \left[ \int_1^x \ln(t) dt \right]$$

$$\frac{d}{dx} \left[ \int_1^{x^2} \ln(t) dt \right]$$

- 2) [5pts each] Compute the integrals:

$$\int_0^{\pi/6} \tan(2\theta) d\theta.$$

$$\int_0^{\pi/4} \sec^2(\theta) d\theta.$$

$$\int 2^t dt$$

$$\int_0^{\pi} \sin^2 3x dx$$

- 3) Find the volume of the solid that is obtained when the region under the curve  $y = \sqrt{x}$  over the interval  $[1,4]$  is revolved around the  $x$ -axis.

- 4) Find all values of  $x^*$  in  $[-3,1]$  that satisfy the Mean-Value Theorem for  $f(x) = |x|$ .

- 5) Use left-hand endpoints, with  $n=3$ , to approximate the area under  $f(x) = 1/x$  on  $[1,4]$ .

- 6) Find the area between the parabolas,  $y = 2x - x^2$  and  $y = 2x^2 - 4x$ .

- 7) Find the displacement and the distance travelled, given  $a(t) = t - 2$ ,  $v_0 = 0$ ,  $1 \leq t \leq 5$ .

- 8) Choose ONE. Explain thoroughly. You can continue on the back.

A) State and prove the FTC, Part 1, about  $\int_a^b f(x) dx$ .

B) State and prove the formula for  $1 + 2 + \dots + n$ .

C) State and justify the volume formula used in the disk method [include a picture, the volume of a disk, a Riemann sum, a limit, etc].

**Answers/Remarks:** The average among the top 25 scores was 62/100. That's about 5 points below expectations. For this test, the A's start at 76, the B's are from 66 to 75, the C's are from 56 to 65, etc. If you scored in the low 50's, you might still pass this course with a C, but you will have to work very hard at it. But below a 45/100, I think you should drop the course. The best grades were on problems 3 and 6; the worst were on 4 and 7 (and 1 maybe).

1a) Telescoping:  $1/20 - 1 = -19/20$

1b) Geometric: The hint was to remind you of the HW you did, and help you get to  $S = (a - ar^{n+1})/(1 - r) = 2^{21} - 1$  (you set  $a = 1$  and  $r = 2$  and  $n = 20$ ).

1c)  $\ln(x)$  by FTC, Part 2.

1d)  $2x \ln(x^2)$  using 1c and the Chain Rule.

2a) You can use  $u = \cos(2\theta)$ . I used  $u = 2\theta$  and got  $(1/2) \ln |\sec u| \big|_0^{\pi/3} = (\ln 2)/2$ .

2b)  $\tan(\theta) \big|_0^{\pi/4} = 1$

2c)  $2^t / \ln(2) + C$ .

2d)  $\int_0^\pi (1 - \cos 6x)/2 \, dx = \pi/2$

3) With the disk method,  $V = \int_1^4 \pi(\sqrt{x})^2 \, dx = 15\pi/2$ . Most people got this one.

4) Set  $|x^*| = \frac{1}{1-(-3)} \int_{-3}^1 |x| \, dx = 5/4$  and get  $x^* = -5/4$  (since  $5/4 \notin [-3, 1]$ ).

Why were the results were so bad on this one (average grade = approx 3/10)? It is similar to HW problem 6.6.65, and the  $|x|$  did not seem to be the problem for most people. Suggestion: pay close attention to the *theorems and definitions* in a math course. They contain the main principles in very compact form. And, if a theorem has a name, it is probably twice as important.

5) Draw a picture and compute the areas of three rectangles:  $1+1/2+1/3 = 11/6$ . Notation such as  $\sum$  or  $f(x_k^*)$  is optional (since we are not going to take a limit).

6)  $\int_0^2 (2x - x^2) - (2x^2 - 4x) \, dx = 4$ . A picture + a sanity check are highly recommended here. 'Insane' answers like -14 got less partial credit than answers like 5 (wrong, but sane).

7)  $v(t) = \int a = t^2/2 - 2t$  (with  $C = v_0 = 0$ ). Displacement =  $\int_1^5 v(t) \, dt = t^3/6 - t^2 \big|_1^5 = 124/6 - 24 = -10/3$  Distance travelled =  $\int_1^5 |v(t)| \, dt = \int_1^4 -v \, dt + \int_4^5 v \, dt = -t^3/6 + t^2 \big|_1^4 + t^3/6 - t^2 \big|_4^5 = \text{etc.}$  The most common mistake here was to confuse  $a$  with  $v$ .

8) See the textbook or lecture notes. Most people chose the FTC and did OK. I took off points mainly when important words were missing (continuous, MVT, telescoping, etc).