

Problems 4-8 are worth 10 points each.

1) [20 points; 4 each] Short calculations. Notice that the second sum starts at 11.

$$\sum_{k=1}^{20} k^2 =$$

$$\sum_{k=11}^{30} k =$$

$$\sum_{k=1}^{2007} \left(\frac{1}{k} - \frac{1}{k+1} \right) =$$

$$\frac{d}{dx} \left[\int_3^x \ln(t) dt \right]$$

$$\frac{d}{dx} \left[\int_1^{x^2} \ln(t) dt \right]$$

2) [15pts] Compute these integrals:

$$\int_0^{\pi/4} \sec^2(\theta) d\theta.$$

$$\int_0^3 |(x-2)(x+1)| dx$$

$$\int_0^{\pi} \sin^2 3x dx$$

3) [15pts] Answer True or False:

In uniformly accelerated motion, velocity is constant.

For continuous functions, $\int_a^b f(x)g(x) dx = \left(\int_a^b f(x) dx \right) \left(\int_a^b g(x) dx \right)$.

If we used only regular partitions of $[a,b]$, then the phrase “ $\max \Delta x_k \rightarrow 0$ ” would be equivalent to “ $n \rightarrow +\infty$ ”.

If f is bounded on $[a,b]$ it is integrable there.

If we use the Right Endpoint Rule to approximate $\int_1^2 \frac{1}{t} dt$, with $n = 100$, our estimate will be too small.

4) Find all values of x^* that satisfy the Mean-Value Theorem for Integrals, for $f(x) = |2x|$ on the interval $[-1,3]$.

5) Use left-hand endpoints in a regular partition, with $n=3$, to approximate the area under $f(x) = x^2 + 1$ on $[1, 4]$.

6) Evaluate the limit by interpreting it as a Riemann sum. Hint: You might factor out n^2 from the denominator first.

$$\lim_{n \rightarrow +\infty} \sum_{k=1}^n \frac{n}{n^2+k^2}$$

7) Find the displacement and the distance travelled, given $v(t) = \cos(t)$, $0 \leq t \leq 3\pi/2$.

8) Choose ONE. Explain thoroughly. You can continue on the back, if needed.

A) State and prove the FTC, Part 1, about $\int_a^b f(x) dx$.

B) State and prove the formula for $1 + 2 + \dots + n$ when n is even.

C) State and prove the FTC, Part 2 [you may use the M.V.Thm without proving it].

Remarks and Answers: The average was about 63/100, normal but a little low. The worst scores, by far, were on problem 6, but problem 4 was a bit low too. The others had fairly high average results. I gave you several choices in problem 8 this time, but sometimes my proof questions have fewer options. Prepare for most of the advertised proofs, if not all.

Unofficial Scale: A's 76-100, B's 66-75, C's 56-65, D's 46-55.

1a) $\frac{20 \cdot 21 \cdot 41}{6} = 2870$. You did not have to simplify fractions like this.

1b) $\sum_{k=1}^{30} k - \sum_{k=1}^{10} k = \frac{30 \cdot 31}{2} - \frac{10 \cdot 11}{2} = 410$. Another good method is to substitute $j = k - 10$, and get $\sum_{j=1}^{20} j + 10 = \frac{20 \cdot 21}{2} + 200 = 410$.

1c) $1 - 1/2008$. It telescopes - write out several terms, and cancel, to be sure you include the correct ones in your answer.

1d) $\ln(x)$ by FTC 2.

1e) $2x \ln(x^2)$ by FTC 2, and the Chain Rule.

2) Notice that all the functions in 2) and 4) are positive. You should not get negative answers (not even for the pieces that arise in 2c and 4). If you do, try again!

2a) $\tan(x) \Big|_0^{\pi/4} = 1$

2b) $31/6$. Split at 2, $\int_0^2(-f) + \int_2^3(+f) = 10/3 + (-3/2 + 10/3) = 31/6$

2c) $\int_0^\pi (1 - \cos(6x))/2 dx = \pi/2$.

3) FFTFT

4) $1 + 9 = \int_{-1}^3 |2x| dx = |2x^*| \cdot (3 - (-1))$, so $x^* = 5/4$.

5) 17. The cleanest way is to draw a picture with three rectangles. Areas = $2+5+10=17$.

It is OK to compute $\Delta x = 3/3 = 1$, $x_k^* = 1 + (k-1)1 = k$, etc, but that notation is needed mainly for harder problems involving a limit or a larger value of n (and for more abstract purposes, such as definitions and proofs). Likewise, the integral in problem 4) is easy to solve with a picture + geometry (but the FTC is also OK). Fancy methods tend to produce more errors, especially if you do not do ‘sanity checks’.

6) Rewrite the sum as $\sum \frac{1}{n} \cdot \frac{1}{1+(k/n)^2}$ (following the hint). Set $x_k^* = k/n$, $a = 0$, $b = 1$, and $\Delta x = 1/n$. The limit is $\int_0^1 \frac{1}{1+x^2} dx = \tan^{-1}(x)|_0^1 = \pi/4$.

Out of curiosity, I used this result to approximate π . I set $n=20$, and used scilab software to compute the Riemann Sum, which gave 0.772 (which should be $\approx \pi/4$). Multiplying by 4, we get $\pi \approx 3.09$. I also tried with this $n=10$, and got 3.03. It doesn't seem to be a very efficient/accurate method.

The grades were extremely low on this one, but it is exercise 6.6.76 (plus a hint), and it is similar to a problem from HW 1; 6.6.75.

7) Displacement = $\int_0^{3\pi/2} \cos(x) dx = -1$. Distance travelled = $\int_0^{3\pi/2} |\cos(x)| dx = 3$ (split the interval at $\pi/2$ to compute this). If you relied on a graph of $\cos(x)$ and the relationship with area, you should explain briefly and include the basic area calculation (such as $\int_0^{\pi/2} \cos(x) dx = 1$) that you based your reasoning on.

8) See the text or lectures notes. When stating a theorem, remember to include the hypotheses. In FTC1, for example, it is important that $F' = f$. When proving one, be sure to explain the main ideas; mention any definitions or previous theorems that you use.