Exam II — March 6, 2006
MAC 2312 and Key — S Hudson

Show all your work. Use the space provided, or leave a note. Don't use a calculator or your own extra paper. Use these formulas as needed:

$\int \sin^n x \; dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \; dx$
$\int u\sqrt{a+bu} \; du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$
$\int \frac{u \; du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$

1) (10pt) Find the arc length of the parametric curve $x = t^3/3$, $y = t^2/2$, with $0 \le t \le 1$.

2) (10pt) A rocket weighing 3 tons is filled with 40 tons of liquid fuel. In the initial part of the flight, fuel is burned off at a constant rate of 2 tons per 1000 ft of vertical height. Find an integral for the work required to lift the rocket to 3000 ft (you do not have to compute the integral).

3) (40pts) Compute each one:

a) $\int xe^{-x} \; dx$

b) $\int \sin^2(x)\cos^2(x) \; dx =$

c) $\int \ln(2x) \; dx =$

d) $\int x\sqrt{2x-3} \; dx$

e) $\int_{-2}^{2} \frac{1}{4+x^2} \; dx$

4) (10pts) a) Write out the form of the Partial Fraction decomposition (but do not compute the coefficients, A, etc):

$\frac{1-x^2}{x^3(x^2+1)} =$

b) For this decomposition, find the coefficients:

$\frac{3x-1}{x^2-3x-4} = \frac{A}{x-4} + \frac{B}{x+1}$

5) (10pt) Use the Shell method to find the volume, when the region bounded by these curves is revolved around the y axis: $y = x$, $y = x^2$. Note that the curves intersect at (0,0) and (1,1).

6) (10pt) Answer True or False:

If a region revolves around the x-axis, the Shell Method uses a dy integral.

If a region revolves around the y-axis, the Disk Method uses a dy integral.

The definition of Surface Area in Ch 7.5 is $S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx$.

For $f(x) = x^3 + x$ on (0,2), $L_8 < R_5$. [Left and Right Endpoint rule]

For $f(x) = x^3 + x$ on (0,2) $T_8 < M_5$. [Trapezoid and Midpoint rule]

7) (10pts) Choose ONE, and explain thoroughly:

a) Explain the integral formula used in the Shell Method. Include: a picture, a limit, a sum, the volume of a shell, and explanation.

b) Explain [as in 7a] the formula used for Arc Length.

c) Prove the reduction formula at the top of page 1.

BONUS: (5 pts) Find the volume when the region bounded by $y = x^3$, $y = 1$ and $x = 0$ is revolved around the line $y = 1$.

Remarks: The average was about 66/100 [among the top 25 grades] , just slightly under expectations, so use the scale on the syllabus for this test. I think the test was not too hard. Most people scored over 70 per cent on problems 3 to 6. Problems 1 and 2 had worse results, but they were assigned HW problems, which everyone should have been ready for. Also, the "proofs" on problem 7 generally did not seem well-prepared. Apparently, some people prepared only for the easy IBP proof, though I always suggest that you learn *most* of the assigned proofs (if not all).

I averaged your two exams scores and wrote it in black at the top of page one, along with your approximate letter grade. For that, I adjusted the scale on the syllabus 5 points.

Answers:

1) $L = \int_0^1 \sqrt{(t^2)^2 + t^2}\ dt = \int_0^1 t\sqrt{t^2+1}\ dt = \frac{1}{2}\int_1^2 u^{1/2}\ du = \frac{1}{3}(2^{3/2} - 1) \approx$ 2/3. The last step is useful (with a rough graph) for a sanity check; but it's optional. The scores on this problem were low considering this was *the assigned HW problem, 7.4.9.* If you forgot the formula for L with parametric curves, you could eliminate t.

2) Draw a picture of the rocket's path (the positive y-axis). Split it into segments of length $\triangle y$, which are the distances in the $F = Wd$ formula. $F =$ the weight, which starts at 43, but decreases by 2/1000 tons per ft. So, $F(y) = 43 - 2y/1000$ and $W = \int_0^{3000} 43 - y/500\ dy$. Again - this was *an assigned HW problem, 7.7.21.* If you only studied water tank problems, take a lesson from this !

3) I graded each part out of 5 points, but they should be worth 8 each. So, I multiplied your total by 1.6 at the top of your page 2. For example, if you got 4 out of 5 right, you should see "20 → 32" at the top.

3a) $-xe^{-x} - e^{-x} + C$ (use IBP)

3b) There are several correct methods and answers (which made it quite hard to grade!). You can use the reduction formula on pg 1: $\int \sin^2 x - \sin^4 x\ dx = [-\frac{1}{2}\sin x\cos x + x/2] + \frac{1}{4}\sin^3 x\cos x$ - $\frac{3}{4}[-\frac{1}{2}\sin x\cos x + x/2] + C$.

Or, $\frac{1}{4}\int(1 - \cos 2x)(1 + \cos 2x)\ dx = \ldots = x/8 - \frac{1}{32}\sin 4x + C$. Other answers are OK because of trig identities (for example, $2\sin x\cos x = \sin 2x$).

3c) $x\ln|2x| - x + C$ (from IBP)

3d) Use the formula on page 1 (like a short table of integrals) for an easy 5 points: $\frac{1}{5}(x+1)(2x-3)^{3/2} + C$. Or, you can use a sub: $u = 2x - 3$ and $x = (u+3)/2$, or use IBP.

3e) $\pi/4$ (Set $x = 2\tan\theta$).

4a) $A/x + B/x^2 + C/x^3 + (Dx + E)/(x^2 + 1)$

4b) $A = 11/5$ and $B = 4/5$

5) $\int_0^1 2\pi x(x - x^2)\ dx = \pi/6$

6) TTTTF

7) See the textbook.

Bonus) The Disk Method leads to $\int_0^1 \pi(1 - x^3)^2\ dx = 9\pi/14$.